M.Sc. Examination 2017

Semester-I

Statistics

Course : MSC-11

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer any four questions

1. (a) Give two definitions of generalized inverse of a matrix. Show that these definitions are equivalent.

   (b) Prove that the general solution of the system of homogeneous equations $A\boldsymbol{x} = 0$ can be expressed as $\hat{\boldsymbol{x}} = (I - H)\boldsymbol{z}$, where $\boldsymbol{z}$ is any arbitrary vector and $H = A^{-}A$. 2+4+4

2. (a) Let A be a matrix of order $m \times n$ $(m \leq n)$ such that $\text{rank}(A) = r$. Suppose it is possible to partition A as

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

   where the matrices A_{11}, A_{12}, A_{21} and A_{22} are of orders $r \times r$, $r \times (n-r)$, $(m-r) \times r$ and $(m-r) \times (n-r)$ respectively. Also $\text{rank}(A_{11}) = r$. Find the generalized inverse of A.

   (b) Find the eigenvalues of the $n \times n$ matrix with all diagonal entries equal to a and all the remaining entries equal to b. 6+4

3. (a) Obtain a set of necessary and sufficient conditions for positive definiteness of a quadratic form.

   (b) If A be a positive definite matrix, then show that there exists a non-singular matrix B such that $A = BB'$. 5+5

4. (a) State and prove Gauss-Markov theorem.

   (b) Consider the model

$$\begin{aligned} y_1 &= \beta_1 + \beta_2 + \epsilon_1 \\ y_2 &= \beta_1 + \beta_3 + \epsilon_2 \\ y_3 &= \beta_1 + \beta_2 + \epsilon_3 \end{aligned}$$

   Obtain a necessary and sufficient condition for $\lambda_1\beta_1 - \lambda_2\beta_2 + \lambda_3\beta_2$ to be estimable. 6+4

P. T. O

5. (a) Define estimation space and error space. Prove that a linear function of observations belongs to error space if and only if the coefficient vector is orthogonal to the columns of X.

   (b) Show that the covariance between any linear function belonging to the error space and any BLUE is zero. 6+4

6. (a) With reference to the model $\boldsymbol{y} = X\beta + e$, where $e \sim N_n(0, \Sigma)$ and X is of full column rank, describe the likelihood ratio test for $H_0 : L\beta = 0$ where the matrix L is of order $r \times n$ and $\text{rank}(L) = r$.

   (b) With reference to pth degree polynomial regression model, obtain the test statistic for testing H_0 : the pth coefficient is zero. 6+4

M.Sc Semester-I Examination, 2017

Statistics

Course : MSC-12

(Regression Analysis)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer any four questions from the following

1. Consider the simple linear regression model : $y_i = \beta_0 + \beta_1 x_i + \in_i$, with $E(\in_i) = 0, \text{var}(\in_i) = \sigma^2$ i_2=1,2,…n and cov $(\in_i, \in_{i'}) = 0, i \neq i'$.

   (a) Show that $Var(\hat{\beta}_1) = \dfrac{\sigma^2}{S_{xx}}, Var(\hat{\beta}_0) = \sigma^2\left(\dfrac{1}{n} + \dfrac{\bar{x}^2}{S_{xx}}\right)$, where $S_{xx} = \sum (x_i - \bar{x})^2$

   (b) $E(MS_{\text{Res}}) = \sigma^2$

   (c) $\sum x_i e_i = 0$ 4+4+2=10

2. (a) Consider the multiple linear regression model $\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{\in}$. Show that the OLS estimator of β is BLUE. 6

   (b) What do you mean by hidden extrapolation in multiple regression? Explain 4

3. Write short notes on :

   PRESS residuals, Partial regression plot, Residual vs. filled values plot for detecting model inadequacies, Detection and treatment of outlier. 2.5×4

4. (a) Discuss an analytical procedure for determining the form of the transformation on the regressor variables. 5

   (b) Consider the model $\underset{\sim}{y} = X\underset{\sim}{\beta} + \underset{\sim}{\in}$, where $E(\underset{\sim}{\in}) = \underset{\sim}{0}$ Var $(\underset{\sim}{\in}) = \sigma^2 V$ with known matrix $V_{n\times n}$.

   Show that the least squares estimator of $\underset{\sim}{\beta}$ is $(X^1V^{-1}X)^{-1}X^1V^{-1}\underset{\sim}{y}$. 5

5. (a) What do you mean by leverage and influence point? How will you detect these points? Discuss. 2+4=6

   (b) Define a cubic spline and a linear spline regression model with k-knot points. 4

6. (a) State and discuss some methods for dealing with multicollinearity. 5

   (b) Describe a method of estimating the parameters in Logistic Regression model. 5

M.Sc Semester-I Examination, 2017
Statistics
Course : MSC-13
(Stochastic Process and Distribution Theory)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Choosing two questions from each of the group answer four question

Group - A

1. (a) Prove that transition probability matrix of any order is stochastic .
   (b) Under Markovian set up, what do you mean by persistent state? Prove that a state j is persistent iff
   $\sum_n p_{jj}^{(n)} = \infty$ where $p_{jj}^{(n)}$ bears the usual meaning.
   (c) Show that for a stationary Markov chain, unconditional probability distribution at initial stage is same as that of n th stage. 3+4+3=10
2. (a) Show that for a symmetric simple random walk model with state space $\{0, \pm 1, \pm 2, ...\}$ all states are null recurrent.
   (b) What are transition densities in the context of continuous time Markov Chain? Based on these densities establish Feller-Kolmogorov backward and forward equations. 5+(2+3)=10
3. (a) If the intervals between two successive occurrences of an event E are independently distributed with a common exponential distribution with mean $\frac{1}{\lambda}$, show that the flow of events forms a Poisson distribution with rate λ.
   (b) Suppose for a Markov process interarrival time distributions are rectangular R(0,1). Find the renewal function associated with that process. 6+4=10
4. (a) Construct Yule-Furry birth process, clearly stating the necessary assumptions to build the model.
   (b) Let N(t) denote the number of renewals by time t for a process with i.i.d. interarrival time variables $X_1, X_2, ..., X_n$ with $E(X_n) = \mu < \infty$.
   Show that
   $$\frac{N(t)}{t} \to \frac{1}{\mu} \text{ as } t \to \infty.$$
   6+4=10

Group – B

5. (a) If $t_n(\lambda)$ denotes the non-central t variable with degress of freedom n and noncentrality parameter λ then prove that $P(t_n(\lambda) \le t) = 1 - P(t_n(-\lambda) \le -t)$.
   (b) Prove that $P(\chi_n^2(\lambda) \le x) = P(X_1 - X_2 \ge \frac{n}{2})$, where X_1 ~ Poisson $\left(\frac{x}{2}\right)$ and $X_2 \sim$Poisson $\left(\frac{\lambda}{2}\right)$ independently and of $\chi_n^2(\lambda)$ denotes the noncentral χ^2 variable with degrees freedom n and non-centrality parameter λ.

P.T.O.

(c) If $X_{n \cdot p}$ is a data matrix from $N_p(\mu, \Sigma)$, then state a necessary and sufficient condition for Y= AXB and and Z = CXD to be independent. 3+5+2=10

6. (a) Under Gauss-Makkov set up, prove that the error sum of square and the regression sum of squares are independently distributed.

(b) Let X_1, X_2,...,X_n be a random sample of size n from $N_4(\mu, \Sigma)$ distribution, where $\underline{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$ and Σ is unknown, Then derive the MLE of $\underline{\mu}$ subject to the following conditions

$$\mu_1 - \mu_2 + \mu_3 = 0$$
$$\mu_2 - \mu_3 + \mu_4 = 1$$
$$\mu_1 + \mu_2 + \mu_3 = 4$$

You have to simplify your answer as much as possible. 3+7=10

7. Prove that $\dfrac{D_p^2 - D_k^2}{m + Dk^2} \sim \dfrac{p-k}{m-k+1} F_{p-k}, m-p+1$ and is independent of D_k^2 if $\underset{\sim}{\mu}_{2.1} = \underset{\sim}{0}$, where D_p^2 is the p dimension at sample Mahalanobis distance. 10

8. (a) Prove that under Gauss-Markov set up, $\underline{Y}'A\underline{Y}$ follows χ^2 distribution if A is independent.

(b) Let $\underset{\sim}{X}_1, \underset{\sim}{X}_2, \ldots, \underset{\sim}{X}_n$ $\underset{\sim}{\text{i.i.d}}$ $Np \sim (\underline{\mu}, \Sigma)$, where

$$\Sigma_{p \times p} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$

Derive the likelihood ratio test for testing

$H_0 : \Sigma_{12} = 0$. 3+7=10

M.Sc Semester-I Examination, 2017
Statistics
Course : MSC-14
(Statistical Inference)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer any four questions from the following

1. (a) State and prove Neyman-Fisher factorization theorem (discrete part only).

   (b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\mu, 1)$ with unknown $(-\infty < \mu < \infty)$. Find an unbiased estimator of μ^2 whose variance attains the Bhattacharya second lower bound. 4+6=10

2. (a) Define uniformly minimum variance unbiased estimator (UMVUE) of a real-valued function $g(\theta)$ of an unknown parameter θ. State and prove a necessary and sufficient condition for an estimator to $\delta(x)$ be UMVUE of its expectation $g(\theta)$.

   (b) Let $(X_1, X_2, \ldots X_n)$ be a random sample of size n drawn from Bernoulli population with parameter π. Find the UMVUE of $g(\pi) = 1 + n\pi + \frac{n(n-1)}{2}\pi^2$. 5+5=10

3. (a) Distinguish between probability mass (or density) function and the likelihood function. Explain the method of maximum likelihood estimation.

   (b) Suppose $(X_1, X_2, \ldots, X_n)$ is a random sample of size n from a distribution with cdf

   $$F(x \mid \alpha, \beta) = 0 \text{ if } x < 0$$
   $$= \left(\frac{x}{\beta}\right)^{\alpha} \text{ if } 0 \le x \le \beta$$
   $$= 1 \text{ if } x > \beta,$$

   Where the parameters $\alpha(>0)$ and $\beta(>0)$ are unknown. Obtain the maximum likelihood estimators of α and β. 6+4=10

4. (a) When is a family of distributions said to be one parameter exponential?

   (b) Derive necessary results to show how a complete sufficient statistic can be identified from the form of this distribution.

   (c) Show that a necessary and sufficient condition for a distribution to admit an unbiased estimator with variance attaining Cramer-Rao lower bound is that it is of the form in (a). 2+4+4=10

5. (a) Define U-statistic. Find the limiting form of the variance of it.

   (b) State the result regarding the asymptotic distribution of U-statistic with conditions, if any. Also find the asymptotic distribution of Wilcoxon's signed rank statistic. 6+4=10

6. (a) Show that no unbiased estimator of a real parameter can be Bayes estimator under squared error loss.

   (b) Suppose $X_i; i = 1(1)n$ follows $N(\mu, 1)$. The prior distribution of μ is $N(0, \sigma^2), \sigma^2 > 0$. Find Bayes estimator of μ under squared error loss. 4+6=10

P.T.O.

7. Write short notes on **any two** of the following : 5+5=10
   (a) MVUE and Method of Covariance.
   (b) Minimum Chi-square method of estimation
   (c) Measure of association between two variables.
   (d) Risk function and bayes risk.

M.Sc. Semester-I Examination 2017
Statistics (Practical)
Paper: MSC - 15

Time: Four Hours **Full Marks: 40**

Questions are of value as indicated in the margin

1) Find the g-inverse of the following matrix

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 & 1 \end{pmatrix}$$ **5**

2) Consider the following quadratic form

$$Q = 9x_1^2 + 7x_2^2 + 3x_3^2 - 2x_1x_2 + 4x_1x_3 + 6x_2x_3$$

Identify the nature of the quadratic form. **3**

3) Following data represent a random sample of size from the Cauchy population with the probability density function $f(x,\theta) = \frac{1}{\pi} \cdot \frac{1}{1+(x-\theta)^2}$; $-\infty < x, \theta < \infty$. Find out the MLE of θ. The observations are 3.7807, 2.9597, 5.2043, 4.8993, 2.6784, 4.9557, 4.9637, 3.4996, 3.1674.

Without assuming any distribution, find out nonparametric estimate of mean and variance functional. **5**

4) For doubly genetic data with some value of π, for both parents, following distribution is obtained:

	D1D2	D1R2	D2R1	R1R2
Frequency:	191	35	34	27
Probability:	$\frac{2+p}{4}$	$\frac{1-p}{4}$	$\frac{1-p}{4}$	$\frac{p}{4}$

where $p = (1-\pi)^2$. Find Maximum Likelihood Estimate of π and estimate its standard error. **5**

5) Consider the data on National Football league as follows:

	y	x1	x2	x3	x4	x5	x6	x7	x8	x9
1	10	2113	1985	38.9	64.7	4	868	59.7	2205	1917
2	11	2003	2855	38.8	61.3	3	615	55.0	2096	1575
3	11	2957	1737	40.1	60.0	14	914	65.6	1847	2175
4	13	2285	2905	41.6	45.3	-4	957	61.4	1903	2476
5	10	2971	1666	39.2	53.8	15	836	66.1	1457	1866
6	11	2309	2927	39.7	74.1	8	786	61.0	1848	2339
7	10	2528	2341	38.1	65.4	12	754	66.1	1564	2092
8	11	2147	2737	37.0	78.3	-1	761	58.0	1821	1909
9	4	1689	1414	42.1	47.6	-3	714	57.0	2577	2001
10	2	2566	1838	42.3	54.2	-1	797	58.9	2476	2254
11	7	2363	1480	37.3	48.0	19	984	67.5	1984	2217
12	10	2109	2191	39.5	51.9	6	700	57.2	1917	1758
13	9	2295	2229	37.4	53.6	-5	1037	58.8	1761	2032
14	9	1932	2204	35.1	71.4	3	986	58.6	1709	2025
15	6	2213	2140	38.8	58.3	6	819	59.2	1901	1686
16	5	1722	1730	36.6	52.6	-19	791	54.4	2288	1835
17	5	1498	2072	35.3	59.3	-5	776	49.6	2072	1914
18	5	1873	2929	41.1	55.3	10	789	54.3	2861	2496
19	6	2118	2268	38.2	69.6	6	582	58.7	2411	2670
20	4	1775	1983	39.3	78.3	7	901	51.7	2289	2202
21	3	1904	1792	39.7	38.1	-9	734	61.9	2203	1988
22	3	1929	1606	39.7	68.8	-21	627	52.7	2592	2324
23	4	2080	1492	35.5	68.8	-8	722	57.8	2053	2550
24	10	2301	2835	35.3	74.1	2	683	59.7	1979	2110
25	6	2040	2416	38.7	50.0	0	576	54.9	2048	2628
26	8	2447	1638	39.9	57.1	-8	848	65.3	1786	1776
27	2	1416	2649	37.4	56.3	-22	684	43.8	2876	2524
28	0	1503	1503	39.3	47.0	-9	875	53.5	2560	2241

a. Fit a multiple linear regression model (Model -I) relating the number of games won (y) to the team's passing yardage (x_2), the percentage of rushing plays (x_7), and the opponents' yard rushing (x_8).

b. Construct the analysis-of-variance table and test for significance of regression.

c. Calculate t statistics for testing the hypotheses $H_0: \beta_2 = 0$, $H_0: \beta_7 = 0$ and $H_0: \beta_8 = 0$. What conclusions you can draw about the role the variables x_2, x_7 and x_8 play in the model?

d. Calculate R^2 and R^2_{adj} for this model.

e. Using the partial F test, determine the contribution of x_2 on the model.

f. Remove x_2 and fit the multiple linear regression model (Model -II) relating the number of games won (y) to the percentage of rushing plays (x_7), and the opponents' yard rushing (x_8).

g. Plot the regression plane over the data.

h. Calculate R^2 and R^2_{adj} for Model - II. How do these quantities compare to the values computed for the Model-I?

i. For Model – II, calculate a 95% confidence interval on β_7. Also find a confidence interval on the mean number of games won by a team when $x_7 = 56.0$ and $x_8 = 2100$. **10**

6) The battery voltage drop in a guided missile motor observed over the time of missile flight is shown in the following data :

```
> x
 [1]  0.0  0.5  1.0  1.5  2.0  2.5  3.0  3.5  4.0  4.5  5.0  5.5  6.0  6.5  7.0  7.5  8.0
[18]  8.5  9.0  9.5 10.0 10.5 11.0 11.5 12.0 12.5 13.0 13.5 14.0 14.5 15.0 15.5 16.0 16.5
[35] 17.0 17.5 18.0 18.5 19.0 19.5 20.0
> y
 [1]  8.33  8.23  7.17  7.14  7.31  7.60  7.94  8.30  8.76  8.71  9.71 10.26 10.91 11.67
[15] 11.76 12.81 13.30 13.88 14.59 14.05 14.48 14.92 14.37 14.63 15.18 14.51 14.34 13.81
[29] 13.79 13.05 13.04 12.60 12.05 11.15 11.15 10.14 10.08  9.78  9.80  9.95  9.51
```

Here x_i indicate the time (in seconds) and y_i indicate voltage drop corresponds to the i-th observation.

a. Fit a cubic spline regression model of y on x with two knot points at 6.2 and 13.1.
b. Plot the model over the data.
c. Construct and interpret a plot of residuals versus the predicted response.
d. Again fit a cubic regression model to the data.
e. Discuss which model is better? **7**

7) Practical note book and Viva-voce. **5**

M.Sc Semester-III Examination, 2017

Statistics

Course : MSC-31

(Real Analysis and Measure Theory)

Time : 3 Hours Full Marks : 40

Questions are of value as indicated in the margin

Answer any four questions

1. (a) What do you mean by a field and a σ-field?

   (b) Show that a σ-field is a field. Is the converse true? Discuss.

   (c) What do you mean by a σ-additive set function? 3+5+2=10

2. (a) Discuss monotonicity and countable subadditivity properties of measure.

   (b) If μ be a finite measure defined on $(\Omega, \mathcal{T})$ then show that for $\{A_n\}$ in $\mathcal{T}$,

   $\overline{\lim}\ \mu(A_n) \le \mu(\overline{\lim}\ A_n)$.

   (c) "If X is a random variable then |X| is also a random variable". Discuss whether the statement is true or, not. 3+5+2=10

3. (a) Using the Dominated Convergence Theorem or, otherwise show that if a random variable X is non negative and integrable then $\frac{d}{dt}E(e^{-tX}) = E(-Xe^{-tx}),\ t > 0$.

   (b) If $\phi, \phi_2, ..., \phi_n$ be the characteristic functions corresponding to distribution functions $F_1, F_2, \ldots, F_n$. Then show that $\sum_{j=1}^{n} C_j\phi_j$ is also a characteristic function, where $\sum_{j=1}^{n} C_j = 1$ and $C_j \ge 0, \forall j = 1(1)n$. 5+5=10

4. (a) Drive the characteristic function of the laplace distribution.

   (b) State and prove Borel Cantelli Lemma. 5+5=10

5. (a) Let $\{X_n\}$ be a sequence of independent random variables such that $P\left(X_k = \frac{1}{2^k}\right) = \frac{1}{2}$ and $P\left(X_k = -\frac{1}{2^k}\right) = \frac{1}{2}$. Verify whether Lyapunov's CLT holds for $\{X_n\}$ or not.

   (b) What do you mean by radius of convergence of a power series? Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{n+1}$. 5+5=10

6. (a) Define open set. Show that the intersection of a finite number of open sets is an open set.

   (b) Define cover and subcover of any set. It S be a subset of $\mathbb{R}$ such that every infinite open cover of S has a finite subcover. Then show that S is compact. 5+5=10

7. (a) Discuss the uniform convergence of the sequence of function

   $\left\{\frac{nx}{1+n^2x^2}\right\}$, where $x \in [0,1]$.

   (b) Find $\lim_{x\to 0} \sum_{n=2}^{\infty} \frac{\cos nx}{n(n+1)}$. 5+5=10

M.Sc Semester-III Examination, 2017

Statistics

Course : MSC-32

(Categorical data analysis and Advanced data analysis)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer any four questions

1. (a) Distinguish between prospective and retrospective studies with examples.

   (b) Explain why the odds ratio is a suitable measure for both of these two studies.

   (c) What do you mean by concordant and discordant pairs? Explain with examples. 3+4+3=10

2. (a) Derive the expression of the standard error of sample odds ratio in a 2×2 contingency table.

   (b) Define Pearsonian χ^2 statistic and explain its use in testing independence in a k×l contingency table. 6+4=10

3. (a) What is the drawback of linear model for a binary response ~~model~~.

   (b) In this context, explain the application of generalized linear model.

   (c) Show that the likelihood equations for a generalized linear model (with usual notation) can be written as

$$\Sigma w(y-\mu)\frac{\partial\eta}{\partial\mu}x_j = 0, \text{for each } j.$$ 2+3+5=10

4. (a) Define pseudo – R^2 measure for goodness of fit of a GLM. Find its expression for binary response.

   (b) Write a short note on 'discreate choice model'. (2+2)+6=10

5. (a) Illustrate Gibbs sampling technique with an example.

   (b) Suppose $(X\,|\,\theta) \sim N(\theta,\sigma^2)$; σ^2 known & $\theta \sim \text{cauchy}(\mu,\tau); \mu,\tau$ known. Describe how Gibbs sampling technique can be applied to simulate from the posterior distribution of $\theta\,|\,X$. 4+6=10

6. (a) Briefly explain the 'Acceptance Rejection sampling' method.

   (b) Suppose a mobile company offers a 4-option plan in some regions and a 5-option plan in some other regions. In other words, in some regions, people are asked to choose their favourite plan from 4 & in some regions, they are asked to choose from 5. The data obtained is the number of peoples opting different plans in different regions. Describe how you can estimate your model parameters using EM algorithm in this case. 3+7=10

7. Write notes on (**any two**) : 5×2=10

   (a) Pearson's residual and Anscombe's residual.

   (b) Logistic regression model and the interpretation of its parameters.

   (c) Fisher's Z-transformation.

M.Sc. Examination 2017
Semester- III
Statistics
Course : MSC-33
(Operations Research and Optimization Techniques)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer any four questions

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1. (a) Deduce the balanced equation of a $M/M/c$ queuing model clearly stating necessary assumptions. The notations bear the usual meaning.

   (b) Using the above equation for $M/M/1$ homogeneous model find out steady state probability at the initial time point and hence find expected waiting time in the system.

   5+3+2

2. (a) What do you mean by residual time in a queuing system?

   (b) Establish Pollaczek-Khinchine formula for studying a general queuing process using the concept of residual time.

   (c) What is the probability that the system is empty when a customer arrives?

   2+6+2

3. (a) What do you mean by Hamiltonian Graph?

   (b) For a five-point TSP how many constraints you need to incorporate to wipe out all possible sub tours? Explain it.

   (c) Formulate a 5-point travelling salesman problem.

   2+4+4

4. (a) Give the notion of Linear Programming Problem. With reference to a Linear Programming Problem, discuss (1) feasible solution (2) basic solution (3) basic feasible solution (4) degeneracy.

   (b) A toy company manufactures two types of toys: A and B. Toy of B takes twice as long as to produce as that of Type A, and the company would have time to make a maximum of 2000 per day. The supply of plastic is sufficient to produce 1500 toys per day (both A and B combined). The type B toy requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs. 3.00 and Rs. 5.00 per toy, respectively on Type A and Type B. If the company wants to maximize his profit, then formulate this problem as a Linear Programming Problem. 6+4

P. T. O.

5. (a) Suppose, we have a non-singular matrix $B = (b_1, b_2, \cdots, b_n)$ and B^{-1} is known. Define $B_a = (b_1, \cdots, b_{r-1}, a, b_{r+1}, \cdots, b_n)$. If the columns of B and B_a are viewed as two bases of E^n, discuss how one can obtain B_a^{-1} based on B^{-1}.

(b) Starting from a feasible solution to a Linear Programming Problem, discuss how one can arrive at an optimal basic feasible solution. 6+4

6. Write short notes on the following.

(a) Branch and bound method.

(b) Duality in Linear Programming 5+5

M.Sc Semester-III Examination, 2017
Statistics
Course : MSC-34 (MSS-3)
(Time Series)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer any four questions

1. (a) Define ACF and write down and prove its properties.
   (b) Give an example of two MA(1) processes having the same ACF. 5+5=10

2. Write down ACF of order K for an AR(1) model

$$X_t = 0.7X_{t-1} + \epsilon_t,$$

where $\{\epsilon_t\}$ is a white noise process. Show that AR(1) model can be expressed as a MA process of infinite order. Hence or otherwise discuss the stationarity of the AR(1) process. 4+4=2=10

3. (a) How will you determine the order of an autoregressive process? Explain.
   (b) Derive the Yule-walker equations satisfied by the ACF of an AR(1) process. 5+5=10

4. (a) Describe least square estimation of parameters of the AR(2) process

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \epsilon_t,$$

Where $\{\epsilon_t\}$ is a white noise process with mean 0 and variance σ^2.

   (b) How will you find the choice of MA periods in fitting a time series data? 6+4=10

5. (a) Show that the ARMA(1,1) process

$X_t = 0.5X_{t-1} + \epsilon_t - 0.3\,\epsilon_{t-1}$, where $\{\epsilon_t\}$ is white noise, is stationary and invertible.

   (b) Find the spectral density function of the MA(2) model

$$X_t = 0.5\,\epsilon_{t-2} + 0.8\,\epsilon_{t-1} + \epsilon_t,$$

where $\{\epsilon_t\}$ is a white noise process with mean 0 and variance σ^2. 5+5=10

6. (a) Define power spectrum of a stationary time series X_t.
   (b) Deduce the power spectrum of white noise, AR(1) and MA(1) series.
   (c) What is Markov property? Which of AR(1) and AR(2) follows Markov property? 2+6+2=10

7. (a) Assess invertibility of the MA(2) process

$$X_t = 2 + \epsilon_t - 5\,\epsilon_{t-1} + 6\,\epsilon_{t-2}.$$

   (b) Find the auto-covariances of different order for the MA(2) process

$$X_t = 1 + \epsilon_t - 5\,\epsilon_{t-1} + 6\,\epsilon_{t-2}.$$

   (c) Comment on stationarity of the process

$$12X_t = 10X_{t-1} - 2X_{t-2} + 12\,\epsilon_t - 11\,\epsilon_{t-1} + 2\,\epsilon_{t-2}.$$ 2+6+2=10

Visva Bharati University
M.Sc. Semester III Examination 2017
Subject: Statistics
Paper: MSC-35

Students are asked to save their codes and outputs in a folder with his/her roll number as name.

Full Marks: 40 Time: 4 Hrs.

1. 88 residents of an Indian city, who were interviewed during a sample survey, are classified below as male or female and also as drinkers or non-drinkers of tea. Do these data reveal any association between sex and drinking habit of tea? Use R to perform a suitable test for this and discuss your findings.

	Male	Female
Drink Tea	40	33
Don't drink tea	3	12

5

2. Suppose the random variable X follows Normal distribution with mean θ and variance 1. We assume the prior density to be Cauchy with location parameter 5 and scale parameter 1. Using Gibbs sampling technique, simulate 50 observations from the posterior distribution of θ given X. Also find the Bayes estimate of the parameter θ under squared error loss and absolute error loss. 10

3. Draw a random sample of suitable size from the Beta (4, 6) distribution using acceptance-rejection sampling (Take your proposal density to be $U(0, 1)$). Construct the histogram of the sample and fit a Beta distribution using the same sample. Draw the actual density and the fitted density over the histogram (on the same graph).

7

4. Draw a sample size 20, from a 8:2 mixture of $Beta(4, 6)$ and $Beta(5, 5)$ distribution. Find the bootstrap and jackknife estimate of the standard error of the sample geometric mean, sample harmonic mean, sample median and the sample standard deviation. Take the number of bootstrap replication to be 50. Draw histograms of bootstrap replications in each case. 6

5. Consider the following data set reporting the number of beetles killed after 5 hours

exposure to gaseous carbon disulphide at various concentrations.

Log Dose	No. of Beetles	No. of killed
1.691	59	6
1.724	60	13
1.755	62	18
1.784	56	28
1.811	63	52
1.837	59	53
1.861	62	61
1.884	60	60

Fit logit and probit models to this data and comment on your fit. 7

6. Practical Note Book and Viva-Voce 5

x——— Best of Luck———x

Visva Bharati University
M.Sc Semester III 2017
Date: 12/12/2017
Subject: Statistics :MSC-36

Full Marks: 40 Time:Four hours

1. Solve by using Big-M method the following linear programming problem:
Maximize $z = -2x_1 - x_2$
subject to

$$\begin{aligned} 3x_1 + x_2 &= 3 \\ 4x_1 + 3x_2 &\geq 6 \\ x_1 + 2x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

5

2. Use dual simplex method to solve
Minimize $z = 3x_1 + x_2$, subject to

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ 2x_1 + 3x_2 &\geq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

5

3. Consider a bank with two tellers. An average of 60 customers per hour arrive at the bank and wait in a single line for an idle teller. The average time it takes to serve a customer is 1 min. The bank opens daily at 9.00 a.m. and closes at 4 p.m. Assume that interarrival times and service times are exponential. Determine
   1. The expected number of customers present in the bank.
   2. The expected length of time a customer spends in the bank.
   3. expected total time that any one teller is busy.

6

4. A salesman has to visit five cities A, B, C, D, and E. The distances(in kilometers) are given in the following table.

	A	B	C	D	E
A	-	1	6	8	4
B	7	-	8	5	6
C	6	8	-	9	7
D	8	5	9	-	8
E	4	6	7	8	-

If the salesman starts from city A and has to come back to city A, which route should he select so that total distance traveled become minimum?

6

5. Find your time series data on your machine and now start answering the following. Remember to store all of your results in a folder created on the desktop of your machine.
    (a) Plot the series.
    (b) Comment on stationarity of the raw series.
    (c) Fit an appropriate ARIMA model to the data.
    (d) Perform a 10-step forecast of the fitted model.
    (e) Plot the original as well as the forcasted series along with standard error bands.

1+1+4+3+4

6. Viva Voce + Practical Note Book

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