Internal Examination, 2022

Subject: Statistical Inference-I — Course: MSC-13

Time: 1 hour — Full marks: 10

Answer any TWO of the following questions:

1. Describe Maximum likelihood method of parameter estimation. State its properties.
2. Let $(x_1, x_2, \ldots, x_n)$ be a sample from the exponential distribution with unknown parameter θ of the form $f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}; x, \theta > 0$. Write $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$. Then, if $L(\theta)$ denotes the likelihood function, show that, for $\theta > 0$, $L(\bar{x}) \geq L(\theta)$, and make your comment.
3. Define confidence interval of a real valued parametric function $\gamma(\theta)$. Let $(x_1, x_2, \ldots, x_n)$ be a sample from $N(\mu, \sigma^2)$. Find $100(1-\alpha)\%$ confidence interval for μ.

Internal Examination, 2022

Subject: Statistical Inference-I — Course: MSC-13

Time: 45 minutes — Full marks: 10

Answer any TWO of the following questions:

1. Define a minimal sufficient statistic. If $X_1, X_2, \ldots, X_m$ are distributed as $N(\mu, \sigma_1^2)$ and $X_{m+1}, X_{m+2}, \ldots, X_{m+n}$ are distributed as $N(\mu, \sigma_2^2)$ independently, obtain a minimal sufficient statistic for $(\mu, \sigma_1^2, \sigma_2^2)$.
2. When a family of probability distributions is said to be complete? When is it called boundedly complete? Is a boundedly complete family always complete? Justify your answer.
3. State and prove Neyman-Fisher factorization theorem (discrete part only).

Internal Examination, 2022

Subject: Statistical Inference-I — Course: MSC-13

Time: 45 minutes — Full marks: 10

Answer any TWO of the following questions:

1. Define U-statistic. Find the limiting form of the variance of it.
2. State the result regarding the asymptotic distribution of U-statistic with conditions, if any.
3. Define Kendall's τ and derive the relationship with U-statistic.

Internal Examination, 2022

Subject: Statistical Inference-I — Course: MSC-13

Time: 45 minutes — Full marks: 10

Answer any TWO of the following questions:

1. Suppose $X_i; i = 1(1)n$ follows Bernoulli with parameter θ. The prior distribution of θ is Beta with parameters α and β. Find Bayes estimate of θ under squared error loss.
2. Describe squared error loss function. Absolute error loss function and all-or-nothing loss function. What are the Bayes estimates in these cases? Comment on Bayes estimate of mean of normal distribution.
3. Show that no unbiased estimator of a real parameter can be Bayes estimator under squared error loss.

Internal Examination, 2022

Subject: Statistical Inference-I (Practical) — Course: MSC-15

Time: 1 hour 30 minutes — Full marks: 10

1. Following data represent a random sample of size from the Cauchy population with the probability density function $f(x,\theta) = \frac{1}{\pi} \cdot \frac{1}{1+(x-\theta)^2}; -\infty < x, \theta < \infty$. Find out the MLE of θ. The observations are 3.7708, 2.9957, 5.2043, 4.8993, 2.7468, 4.9557, 4.9367, 3.9649, 3.1674.
   Without assuming any distribution, find out nonparametric estimate of mean and variance functional.
2. For double genetically data with some value of π, for both parents, following distribution is obtained:

	D1D2	D1R2	D2R1	R1R2
Frequency:	191	36	33	27
Probability:	$\frac{2+p}{4}$	$\frac{1-p}{4}$	$\frac{1-p}{4}$	$\frac{p}{4}$

where $p = (1-\pi)^2$. Find Maximum Likelihood Estimate of π and estimate its standard error.