Problem 1: A manufacturer of electric bulbs claims that he has developed a new production process which will increase the mean efficiency (in suitable units) from the present value 9.03. The results obtained from an experiment with 15 bulbs from the new process are given below–

9.29, 9.76, 8.93, 10.15, 12.05, 9.02, 8.69, 12.38, 10.81, 11.25, 9.08, 10.00, 11.47, 10.25, 11.56

⇒ $H_0: \theta_0 = 9.03$

against,

$H_1: \theta_0 > 9.03$

$S > K_\alpha \rightarrow$ rejection rule

$$\sum_{S=K_\alpha+1}^{15} \binom{15}{S}\left(\frac{1}{2}\right)^{15} \le 0.05 \rightarrow \text{condition}$$

$K_\alpha = 12$

$S > K_\alpha$ (reject / critical condition)

$$\sum \binom{15}{S}\left(\frac{1}{2}\right)^{15} \le 0.05$$

K_α is the min value that,

$\therefore K_\alpha = 12$

since, for, $\displaystyle\sum_{S=11}^{15} \binom{15}{S}\left(\frac{1}{2}\right)^{15} = 0.059$

for, $\displaystyle\sum_{S=12}^{15} \binom{15}{S}\left(\frac{1}{2}\right)^{15} = 0.017$

Therefore from trial-error method we find that, $K_\alpha = 12$

we reject if $S > 12$

Interpretation:- Since, $\displaystyle\sum_{S=12}^{15} \binom{15}{S}\left(\frac{1}{2}\right)^{15} < 0.05$ and

$\displaystyle\sum_{S=11}^{15} \binom{15}{S}\left(\frac{1}{2}\right)^{15} > 0.05$, we take, $K_\alpha + 1 = 12$ and $K_\alpha = 11$

$\therefore$ We reject if $S > 11$

Problem 2: Below are given the marks obtained by a group of 20 students in a subject in a college test and in the subsequent public examinaⁿ. Test at 1% level whether the group has improved its net performance from the college test to the public exam by using,

i) Signed Test
ii) Signed rank test (critical value = 43)

Serial No.	Marks in college Test (X_i)	Marks in public exam (Y_i)
1	183	133
2	175	192
3	134	170
4	170	164
5	183	199
6	167	160
7	120	168
8	175	158
9	126	162
10	187	176
11	123	126
12	121	141
13	175	103
14	133	126
15	144	146
16	109	155
17	165	162
18	144	161
19	164	182
20	125	119

i	$Y_i - X_i = D_i$	$\lvert D_i \rvert$	$\text{Rank}(\lvert D_i \rvert)$	Z_i
1	-50	50	19	0
2	18	18	12.5	1
3	36	36	15.5	1
4	-6	6	4.5	0
5	16	16	9	1
6	-7	7	6.5	0
7	48	48	18	1
8	-17	17	10.5	0
9	36	36	15.5	1
10	-11	11	8	0
11	3	3	1.5	1
12	20	20	14	1
13	-72	72	20	0
14	-7	7	6.5	0
15	2	2	1	1
16	46	46	17	1
17	-3	3	1.5	0
18	17	17	10.5	1
19	18	18	12.5	1
20	-6	6	4.5	0

Signed-test statistic:-

S = no. of positive terms among differences is 11.

Signed rank test:-

$$W^+ = 127, \quad W^- = \frac{n(n+1)}{2} - W^+$$

Test statistic $= \min(W^+, W^-) = 210 - 127 = 83$

$= 83$

critical value is $43 < \min(W^+, W^-)$

Two-sample locaⁿ problem:-

$H_0: F_X(x) = F_Y(x) \ \forall x$ (The pdfⁿs $F_X(\cdot)$ and $F_Y(\cdot)$ are identical)

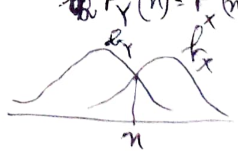
against the alt. H_1: $F_X(x) > F_Y(x)$ (Y is stochastically larger than X)

$F_X(x) < F_Y(x)$ (Y is stochastically smaller than X)

$F_X(x) \neq F_Y(x)$

$(\delta < 0)$

$F_Y(x) = F_X(x$ [illegible]



P-533

18.20) (X) Popular washer : 13 10 9 12 11 10 ~~8~~ 8

(Y) New washer : 10 11 12 13 9 11 14 12 13

$n_1 = 7,\ n_2 = 9$, $n_1 + n_2 = 16$

$N = 16$

$U = 1+1+2+4+6$
$= 14$

8	9	9	10	10	10	11	11	11	12	12	12	13	13	13	14	No. of runs
X	X	Y	X	X	Y	X	Y	Y	X	Y	Y	X	Y	Y	Y	10
	~~X~~		Y	X	X	X	Y	Y	Y	Y	X	X	Y	Y	Y	6
X	Y	X	Y	X	X	Y	X	Y	X	Y	Y	X	Y	Y	Y	12

$\therefore\ W = 14 + \frac{9 \times 10}{2} = 59$

$U = 14$

$U' = 63 - 14$
$= 49$

$U^* = \dfrac{14 - 63/2}{\sqrt{63 \times 17/12}}$
$= -1.852$

From table VIII of Appendix C, we find that for $n_2 = 9$, $n_1 = 7$, for a two tail test at the level 0.02 ~~0.50~~, the critical value is 9. Since 49 is greater than 9, we have no reason to believe that the samples are ~~not~~ drawn from identical distⁿ. Null hypothesis will be accepted.

Run test:

Thus we have $n_1 = 7$, $n_2 = 9$, $N = 16$ and 5 runs of ~~8~~ X's and 5 runs of Y's giving $r = 10$. The critical value of r at the 5% level from the table is 4. Since the observed value (8) is greater than the critical value (4), we accept the null hypothesis (that the popⁿ ~~seare~~ distⁿ are identical) at the 5% level.

⊛ $H_0 :- \delta = 0$ $U < C_\alpha$ For $n_1 = 7$

$H_1 :- \delta > 0$ $U < 9$ $n_2 = 9$

$C_\alpha = 9$

Example :-

The following are the marks secured by two batches of salesman in the final test taken after the completion of training.

Use U-test and Run test for the null hypothesis that the samples are drawn from identical distⁿ against the alt. that the distⁿs differ in locaⁿ only

Batch A (X)	26	27	31	26	19	21	20	25	30
Batch B (Y)	23	28	26	24	22	19			

⇒ $n_1 = 9, \ n_2 = 6$

$N = 15$

19	19	20	21	22	23	24	25	26	26	26	27	28	30	31	No.
X	Y	X	X	Y	Y	Y	X	Y	X	X	X	Y	X	X	9
Y	X	X	X	Y	Y	Y	X	Y	X	X	X	Y	X	X	8
X	Y	X	X	Y	Y	Y	X	X	Y	X	X	Y	X	X	9
Y	X	X	X	Y	Y	Y	X	X	Y	X	X	Y	X	X	8(4
X	Y	X	X	Y	Y	Y	X	X	X	Y	X	Y	X	X	9(5
Y	X	X	X	Y	Y	Y	X	X	X	Y	X	Y	X	X	8(4

Max $r = 9$, min $r = 8$

critical value at $n_1 = 9, n_2 = 6$, is 4

∴ Null hypothesis will be accepted.

$U = 1 + 1 + 4 + 4 + 4 + 5 + 6 + 6 = 31$

$W = 31 + \frac{6 \times 7}{2} = 52$ $U' = 54 - 31 = 23$

∴ Null hypothesis will be accepted as $23 > 7$

H_1	critical region (for size-α test Asymptotic)
$\delta > 0$	$\dfrac{T - \frac{n_2}{n}}{\sqrt{\frac{n_1 n_2}{4n}}} > \tau_\alpha$
$\delta < 0$	$\dfrac{T - n_2/n}{\sqrt{\frac{n_1 n_2}{4n}}} < -\tau_\alpha$
$\delta \neq 0$	$\left\lvert \dfrac{T - n_2/n}{\sqrt{\frac{n_1 n_2}{4n}}} \right\rvert > \tau_{\alpha/2}$

(*)

19	19	20	21	22	23	24	25	26	26	26	27	28	30	31
X	Y	X	X	Y	Y	Y	X	Y	X	X	X	Y	X	X

$n_1 = 9$
$n_2 = 6$
$N = 13$
median $= 25$

Number of $Y > 25$ is 2.

p-value

$\therefore P(T=0) + P(T=1) + P(T=2)$

$$= \frac{\binom{8}{6}\binom{7}{0}}{\binom{15}{6}} + \frac{\binom{8}{5}\binom{7}{1}}{\binom{15}{6}} + \frac{\binom{8}{4}\binom{7}{2}}{\binom{15}{6}}$$

$$= \frac{\frac{8\times7}{2} + \frac{8\times7\times6}{6}\times7 + \frac{8\times7\times6\times5}{24}\times\frac{7\times6}{2}}{\binom{15}{6}}$$

$$= \frac{28 + 49\times8 + 49\times30}{\binom{15}{6}} = 0.3776$$

$\therefore 2\times0.3776 = 0.7552 > 0.05$ ($\therefore$ Null hypothesis will be accepted)

(x)

8	9	9	10	10	10	11	11	11	12	12	12	13	13	13	14
X	X	Y	X	X	Y	X	Y	Y	X	Y	Y	X	Y	Y	Y

$n_1 = 7$
$n_2 = 9$
$N = 16$

Number of $Y > 11$ is 5

p-value,

$\cancel{P(T=0) + P(T=1) + P(T=2) + P(T=3) + P(T=4) + P(T=5)}$

$\therefore P(T=5) + P(T=6) + P(T=7) + P(T=8)$

$$= \frac{\binom{8}{4}\binom{8}{5} + \binom{8}{3}\binom{8}{6} + \binom{8}{2}\binom{8}{7} + \binom{8}{1}\binom{8}{8}}{\binom{16}{9}}$$

$= 0.5$

$2 \times 0.5 = 1 > 0.05$

$\therefore$ Null hypothesis will be accepted

Wilcoxon's Rank Sum Test:-

Let, $x_1, x_2, \ldots, x_{n_1}$ be iid with dist^n fu^n $F(\cdot)$ and $y_1, y_2, \ldots, y_{n_2}$ be iid with dist^n fu^n $G(\cdot)$ with $G(x) = F(x-\delta)$, $\delta \in \mathbb{R}$.
The samples are drawn independently of each other and $F(\cdot)$ is univariate cont.
Let, $\underset{\sim}{Z} = (X_1, X_2, \ldots X_{n_1}, Y_1, Y_2, \ldots, Y_{n_2})$
Let, R_i be the rank of the ith

Theorem

$$P\left(D_n < \frac{1}{2n} + v\right) = \begin{cases} \int_{\frac{1}{2n}-v}^{\frac{1}{2n}+v} \int_{\frac{1}{3n}-v}^{\frac{1}{3n}+v} \cdots \int_{\frac{2n-1}{2n}-v}^{\frac{2n}{}} f(u_1, u_2, \ldots, u_n)\, du_1 du_2 \ldots du_n & \text{if } 0 < v < \frac{2n-1}{2n} \\ 1 & \text{if } v \geq \frac{2n-1}{2n} \end{cases}$$

provided,
$F_X(\cdot)$ is cont.

Theorem

If $F_X(\cdot)$ is cont., then for every $d > 0$

$$\lim_{n\to\infty} P\left(D_n \leq \frac{d}{\sqrt{n}}\right) = 1 - 2\sum_{i=1}^{\infty} (-1)^{i-1} e^{-2i^2 d^2}$$

Practical 1) The 20 obsⁿˢ below were chosen randomly from $U(0,1)$ distⁿ, recorded to 4 significant figures and rearranged in ascending order of magnitude. Test the null hypothesis that the square root of these numbers also have $U(0,1)$ distⁿ.

0.0123, 0.1039, 0.1954, 0.2621, 0.2802, 0.3217, 0.36
0.3919, 0.4240, 0.4814, 0.5139, 0.5846, 0.6215, 0.6541,
0.6889, 0.7621, 0.8320, 0.8871, 0.9249, 0.9634.

$$\Rightarrow \quad D_n = \sup_x |F_n(x) - F_0(x)|$$

$$= \max_x \left[|F_n(x) - F_0(x)|, |F_n(x-\varepsilon) - F_0(x)| \right]$$

$$F_n(x) = \frac{\text{No. of obs}^n \leq x}{\text{Total no. of obs}^n}$$

H_0:- Samples are from $U(0,1)$

$\therefore \langle F_0(x) = x, \ x \in (0,1) \rangle$

x	$F_n(x)$	$F_0(x)$	$\lvert F_n(x) - F_0(x) \rvert$	$\lvert F_n(x-\varepsilon) - F_0(x) \rvert$
0.1109	0.05	0.1109	0.0609	0.1109
0.3223	0.1	0.3223	0.2223	0.2723
0.4420	0.15	0.4420	0.2920	0.3420
0.5119	0.20	0.5119	0.3119	0.3619
0.5293	0.25	0.5293	0.2793	0.3293
0.5672	0.30	0.5672	0.2672	0.3172
0.6037	0.35	0.6037	0.2537	0.3037
0.6260	0.40	0.6260	0.2260	0.2760
0.6512	0.45	0.6512	0.2012	0.2512
0.6938	0.50	0.6938	0.1938	0.2438
0.7169	0.55	0.7169	0.1669	0.2169
0.7646	0.60	0.7646	0.1646	0.2146
0.7921	0.65	0.7921	0.1421	0.1921
0.8088	0.70	0.8088	0.1088	0.1588
0.8300	0.75	0.8300	0.0800	0.13000
0.8729	0.80	0.8729	0.0729	0.1229
0.9121	0.85	0.9121	0.0621	0.1121
0.9419	0.90	0.9419	0.0419	0.0919
0.9617	0.95	0.9617	0.0117	0.0617
0.9815	1	0.9815	0.0184	0.0315

$D_n = 0.3619$

$\alpha = 0.05$

$n = 20$

If observed $D_n > 0.294$ $(\alpha = 0.05)$ then reject H_0.

Conclusion:- Reject H_0, i.e, the sample is not coming from $U(0,1)$

$X \sim U(0,1)$

$\sqrt{X} \sim ?$

$P(\sqrt{X} \le x)$

$= P(X \le x^2) = x^2$

$\therefore$ The c.d.f of the r.v $Y = \sqrt{X}$

$F_Y(y) = y^2$

P.d.f of y, $f_Y(y) = \frac{d}{dy}(y^2)$

$= 2y, \; 0 < y < 1$

Beta $(2,1)$

2) 1.5, 2.3, 4.2, 7.1, 10.4, 8.4, 9.3, 6.5, 2.5, 4.6.
~~The~~ Test whether the data comes from exponential distⁿ.

⇒ For Exp(θ) distⁿ, $\hat{\theta} = \frac{1}{\bar{x}}$

$F_0(x) = 1 - e^{-\hat{\theta}x}$ to use Kol-Sim- test statistic

Since the parameter θ of the exponential distⁿ is not specified in the problem we take the MLE of θ, $\hat{\theta} = \frac{1}{\bar{x}}$ as an estimate of θ and use the formula

$\bar{x} = 5.68$

$\therefore \hat{\theta} = \frac{1}{5.68} = 0.1761$

x	$F_0(x)$	$F_n(x)$	$\lvert F_n(x) - F_0(x)\rvert$	$\lvert F_n(x-\epsilon) - F_0(x)\rvert$
1.5	0.2321	0.1	0.1321	0.2321
2.3	0.3330	0.2	0.1330	0.2330
~~4.2~~ 2.5	0.3561	0.3	0.0561	0.1561
~~7.1~~ 4.2	0.5227	0.4	0.1227	0.2227
~~10.4~~ 4.6	0.5552	0.5	0.0552	0.1552
~~8.4~~ 6.5	0.6817	0.6	0.0817	0.1817
~~9.3~~ 7.1	0.7136	0.7	0.0136	0.1136
~~6.5~~ 8.4	0.7722	0.8	0.0278	0.0722
~~2.5~~ 9.3	0.8056	0.9	0.0944	0.0056
~~4.6~~ 10.4	0.8398	1	0.1602	0.0602

$D_n = 0.2330$.

$\alpha = 0.05$
$n = 10$
0.409

The two sided Kolmogorov - Smirnov - test statistic, denoted by D_{n_1, n_2} is defined as,

$$D_{n_1, n_2} = \max_{x} |F_{n_1}(x) - G_{n_2}(x)|$$

Since here only the magnitudes of the deviaⁿs are considered, D_{n_1, n_2} is appropriate for a general two sided alternative. The critical region is given by,

$$D_{n_1, n_2} \geq C_\alpha$$

where, C_α is such that,

$$P_{H_0}(D_{n_1, n_2} \geq C_\alpha) \leq \alpha$$

When $n_1, n_2 \to \infty$ in such a way that n_1/n_2 remains constant,

$$\lim_{n_1, n_2 \to \infty} P\left(\sqrt{\frac{n_1 n_2}{n_1+n_2}} D_{n_1, n_2} \leq d\right) = 1 - 2\sum_{i \geq 1}^{\infty} (-1)^{i-1} e^{-2i^2 d^2}$$

Example

Normal $N(0,1)$ X : -1.91, -1.22, -0.96, -0.72, 0.14, 0.82, 1.45, 1.86

Chi-Square χ^2_{18} Y : 4.90, 7.25, 8.04, 14.10, 18.3, 21.21, 23.1, 28.12

Two mutually independent random samples, each of size 8 were generated, first one from the N(0,1) & the second one from the χ^2_{18}. Based on these samples investigate whether the standarized chi-square variable $\frac{\chi^2 - n}{\sqrt{2n}}$ approaches a N(0,1) distⁿ even for moderate d.f.

$\Rightarrow X \to -1.91, -1.22, -0.96, -0.72, 0.14, 0.82, 1.45, 1.86$

$Z\ \cancel{Y} \to -2.183, -1.791, -1.66, -0.65, 0.05, 0.535, 0.85, 1.687$

Table

Combined ordered obs^n (t)	$\#X \le t$	$\#Y \le t$	$F_{n_1}(t)$	$G_{n_2}(t)$	$\lvert F_{n_1}(t) - G_{n_2}(t)\rvert$
-2.183 (Y)	0	1	0	$1/8$	$1/8$
-1.91 (X)	1	1	$1/8$	$1/8$	0
-1.791 (Y)	1	2	$1/8$	$2/8 = 1/4$	$1/8$
-1.66 (Y)	1	3	$1/8$	$3/8$	$2/8 = 1/4$
-1.22 (X)	2	3	$2/8 = 1/4$	$3/8$	$1/8$
-0.96 (X)	3	3	$3/8$	$3/8$	0
-0.72 (X)	4	3	$4/8 = 1/2$	$3/8$	$1/8$
-0.65 (Y)	4	4	$4/8 = 1/2$	$4/8 = 1/2$	0
0.05 (Y)	4	5	$4/8 = 1/2$	$5/8$	$1/8$
0.14 (X)	5	5	$5/8$	$5/8$	0
0.535 (Y)	5	6	$5/8$	$6/8 = 3/4$	$1/8$
0.82 (X)	6	6	$6/8 = 3/4$	$6/8 = 3/4$	0
0.85 (Y)	6	7	$6/8 = 3/4$	$7/8$	$1/8$
1.45 (X)	7	7	$7/8$	$7/8$	0
1.687 (Y)	7	8	$7/8$	1	$1/8$
1.86 (X)	8	8	1	1	0

$P_{H_0}(n_1 n_2 D_{n_1,n_2} \ge 16)$

$\ge P_{H_0}(n_1 n_2 D_{n_1,n_2} \ge 32)$ $\quad \langle P_{H_0}(n_1 n_2 D_{n_1,n_2} \ge 32) = 0.283 \rangle$ From the table.

$= P_{H_0}(64\, D_{n_1,n_2} \ge 16)$

$= P_{H_0}(D_{n_1,n_2} \ge 1/4) \to$ p-value

$$\text{p-value} = P_{H_0}(D_{n_1,n_2} \ge 1/4) = P_{H_0}(n_1 n_2 D_{n_1,n_2} \ge 16)$$

$$\ge P_{H_0}(n_1 n_2 D_{n_1,n_2} \ge 32) = 0.283$$

$$\boxed{\text{p-value} \ge 0.283 > \alpha}$$

accept

the locaⁿ parameters of i^{th} & j^{th} popⁿ differs significantly if,

$$\frac{|\bar{R}_i - \bar{R}_j|}{\left[\frac{N(N+1)}{12}\left(\frac{1}{n_i}+\frac{1}{n_j}\right)\right]^{1/2}} \geq \tau_{\alpha/2}$$

where, $\bar{R}_i = R_i/n_i$ = Avg. rank of the obsⁿs corresponding to group i

If $n_i = N/K \;\forall\, i = 1(1)K$

Then,
$$\frac{|\bar{R}_i - \bar{R}_j|}{\left[\frac{(N+1)K}{6}\right]^{1/2}} \geq \tau_{\alpha/2}$$
is the rejecⁿ criterion

Q. In a comparison of the cleaning acⁿ of four detergents, 20 pieces of white cloth were, 1st soiled with ink, the cloths were then washed under control condiⁿ with 5 pieces washed by each of the detergents. Whiteness readings are shown below:-

Detergent

A	B	C	D
77	74	73	76
81	66	78	85
61	58	57	77
76	63	69	64
69	61	63	80

Test the hypothesis of no difference b/w four brands of detergents regarding the avg. whitness reading after washing it.

combined obs^{n}s in ascending order

57(C) 1	73(C) 11
58(B) 2	74(B) 12
61(A) 3.5	76(A) 13.5
61(B) 3.5	76(D) 13.5
63(B) 5.5	77(A) 15.5
63(C) 5.5	77(D) 15.5
64(D) 7	78(C) 17
66(B) 8	80(D) 18
69(C) 9.5	81(A) 19
69(A) 9.5	85(D) 20

$R_1 = 3.5 + 9.5 + 13.5 + 15.5 + 19$
$= 61$

$R_2 = 2 + 3.5 + 5.5 + \cancel{12}\ 8 + \ldots$
$= 31$

$R_3 = \cancel{\ldots}\ 5.5 + 9.5 + 11 + 1 + 1\ldots$
$= 44$

$R_4 = 7 + 13.5 + 15.5 + 18 + 20$
$= 74$

Now,

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{4} \frac{R_i^2}{n_i} - 3(N+1)$$

$n_i = 5 \ \forall\ i = 1(1)K$

$K = 4 \quad n_i = 5 \ \forall\ i = 1(1)K$

$N = 20$

Reject H_0 if $H > \chi^2_{3, 0.05}$

$$\therefore H = \frac{12}{20 \times 21 \times 5} \times \left[61^2 + 31^2 + 44^2 + 74^2\right] - 3(21)$$

$= 6.109 < \chi^2_{3, 0.05} = 7.815$

Accept H_0.

Note: For $K = 3$, and $n_1, n_2, n_3 \leq 5$

Use a special table of Kruskal-Wallis test.

For, $K > 3$, use χ^2 table.

Q. The following table shows the lifetimes in hours in excess of thousand hours, of the samples of 60W electric light bulbs of 3 different brands.

Brand I	Brand II	Brand III
16	18	26
15	22	31
13	20	24
21	16	30
15	24	24

Test at 5% level the hypothesis that there is no difference b/w the three brands with respect to avg. life time

⇒ combined obsⁿ is ascending order

13 (I) 1	24 (II) 11
15 (I) 2.5	24 (III) 11
15 (I) 2.5	24 (III) 11
16 (I) 4.5	26 (III) 13
16 (II) 4.5	30 (III) 14
18 (II) 6	31 (III) 15
20 (II) 7	
21 (I) 8	
22 (II) 9	

$R_1 = 1 + 2.5 + 2.5 + 4.5 + 8$
$= 18.5$

$R_2 = 4.5 + 6 + 7 + 9 + 11$
$= 37.5$

$R_3 = 120 - 18.5 - 37.5$
$= 64$

Now,

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{3} \frac{R_i^2}{n_i} - 3(N+1)$$

$n_i = 5 \ \forall i = 1(1)4$

$K = 3$

$N = 15$

$$\therefore H = \frac{12}{15 \times 16} \times \frac{1}{5} \times \left[18.5^2 + 37.5^2 + 64^2\right] - 3 \times 16$$

$= 10.445 > 5.780$

Reject H_0.

Pairwise Comparision

we know that,

$$T = \frac{|\bar{R}_i - \bar{R}_j|}{\left[\frac{(N+1)K}{6}\right]^{1/2}} \geq \tau_{\alpha/2} = 1.96$$

$\alpha = 0.05$

$K = 3,$

$N = 15,$

$\bar{R}_1 = 3.7$ $\bar{R}_3 = 12.8$ $|\bar{R}_i - \bar{R}_j| = 3.8, 5.3, 9.1$ $\left(\frac{(N+1)K}{6}\right)^{1/2} = 2.828$

$\bar{R}_2 = 7.5$

⇒

~~$T = 4.020,\ 5.620,\ 9.65 \rightarrow 1.96$~~

$\therefore$ Reject.

There has significe difference.

$T = 1.344 < 1.96 \rightarrow$ Accept

$= 1.874 < 1.96 \rightarrow$ Accept

$= 3.21 > 1.96 \rightarrow$ Reject

Q.) An institute of microbiology is interested in purchasing microscope slides of uniform thickness and needs to choose b/w two different suppliers. Both have the same specifica^n's for the median thickness but they may differ in variability. The institute measures the thickness of random sample of 10 slides from each supplier and reports the data shown below as the deviation from specified median thickness. Which supplier makes slides with a smaller variability in thickness?

Supplier 1: 0.028, 0.029, 0.011, −0.030, 0.017, −0.012, −0.027, −0.018, 0.022, −0.023

Supplier 2: −0.002, 0.016, 0.005, −0.001, 0, 0.008, −0.005, −0.009, 0.001, −0.019.

⇒

$\sum D_{1i} = 0 \quad \forall i = 1(1)10$ $\qquad \sum D_{6i} = 1 \quad \forall i = 1(1)10$

$\sum D_{2i} = 0 \quad \forall i = 1(1)10$ $\qquad \sum D_{7i} = 0 \quad \forall i = 1(1)10$

$\sum D_{3i} = 1 \quad \forall i = 1(1)10$ $\qquad \sum D_{8i} = 1 \quad \forall i = 1(1)10$

$\sum D_{4i} = 0 \quad \forall i = 1(1)10$ $\qquad \sum D_{9i} = 0 \quad \forall i = 1(1)10$

$\sum D_{5i} = 0 \quad \forall i = 1(1)10$ $\qquad \sum D_{10i} = 0 \quad \forall i = 1(1)10$

$\therefore \langle T = 3 \rangle$

$$\therefore \frac{4\sqrt{3}\left(3 - \frac{10 \times 10}{4}\right)}{\sqrt{100 \times 27}} = \boxed{-2.933} < -\tau_\alpha$$

$[\alpha = 0.05,\ \tau_\alpha = 1.96]$

When the alt. hypothesis is, $\boxed{\sigma_X > \sigma_Y}$

or, $\boxed{\theta = \frac{\sigma_X}{\sigma_Y} = 1}$

then the critical region is,

$$\left\{ \begin{array}{l} T - n_1 n_2/4 \leq k_\alpha \text{ or} \\ \dfrac{4\sqrt{3}\left(T - \frac{n_1 n_2}{4}\right)}{\sqrt{n_1 n_2 (N+7)}} < -\tau_\alpha \end{array} \right.$$

Here, $T = 3$, $n_1, n_2 = 10$
$N = n_1 + n_2 = 20$

So, that,
$$\frac{4\sqrt{3}\left(T - \frac{n_1 n_2}{4}\right)}{\sqrt{(n_1 n_2)(N+7)}} = -2.933,$$ For $\alpha = 0.05$, $\tau_\alpha = 1.96$

<Null hypothesis is rejected>

Q) Two potential suppliers of street lighting equipments A & B ~~has~~ presented their bids to the city manager along with the following data as a random sample of life length in months.

A : 35, 66, 58, 83, 71
B : 46, 56, 60, 49

Test whether the life length of suppliers A and B have equal variability assuming ~~there~~ their locations to be equal.

$\Rightarrow$
$\sum_i D_{1i} = 4 \quad \forall i = 1(1)4$
$\sum_i D_{2i} = 0 \quad$ "
$\sum_i D_{3i} = 1 \quad$ "
$\sum_i D_{4i} = 0 \quad$ "
$\sum_i D_{5i} = 0 \quad$ "

$\therefore T = 5$

$$\therefore \frac{4\sqrt{3}\left(5 - \frac{5\times 4}{4}\right)}{\sqrt{20 \times 16}} = 0 < |\tau_{\alpha/2}|$$
$$< \tau_\alpha = [1.96 \text{ at } \alpha = 0.05]$$

$\therefore$ <Null hypothesis is accepted>

When the alt. hypothesis is, $\boxed{\sigma_X \neq \sigma_Y}$

or, $\boxed{\theta = \frac{\sigma_X}{\sigma_Y} \neq 1}$

then the critical region is,
$$\left|\frac{4\sqrt{3}\left(T - \frac{n_1 n_2}{4}\right)}{\sqrt{n_1 n_2 (N+7)}}\right| > \tau_{\alpha/2}$$