Drawing Of Random Samples.

Problem 01 ~ Draw a random sample of 5 villages from a group of 337 villages of West Bengal by (i) SRSWR
(ii) SRSWOR.

Solution ~ i) We shall take three-digited numbers from the table of random numbers. To ensure equal probability for each unit, we shall take the numbers from 001 – 674 (The greatest three digited multiple of 337) and shall ignore the other three-digited numbers. We shall divide the number by 337 and take the remainder.
The remainder, varies from 000 to 336. The remainders 001 to 336 will be taken to correspond to villages with the same numbers, whereas the remainder 000 will correspond to 337th village. Since here the sampling is with replacement a village once selected may be selected again. ($N = 337$, $n = 5$)

Table ~ 01

Showing the selection of a random sample of 5 villages from a group of 337 villages with replacement.

Number taken from the table (Random no. R)	Remainder when divided by 337. R (mod 337)	Serial numbers of the village selected
465	128	128
238	238	238
198	198	198
431	094	94
215	215	215

5 villages with serial number 128, 238, 198, 94 & 215 are selected from a group of 337 villages of west bengal by SRSWR.

ii) Here also, we shall take three-digited numbers from the table of random numbers. To ensure equal probability for each village, we shall take the numbers from 001 to 674 (The greatest three-digited number, which is a multiple of 337) and shall ignore the other three digited numbers. We shall divide the number by 337 and take the remainder. The remainder varies from 000 to 336. The remainder 001 to 336 will be taken to correspond to the villages with the same number, whereas the remainder 000 will correspond to the 337th village. Since the sampling is without replacements, a village once selected cannot be selected again. (N = 337, n = 5)

Table ~ 02

Showing the selection of a Random sample of 5 villages from a group of 337 villages without replacements.

Number taken from the table (Random no. R)	Remainder when divided by 337 R(mod 337)	Serial number of the village selected
023	023	23
522	185	185
472	135	135
004	004	4
334	334	334

5 villages with serial number 23, 185, 135, 4, 334 are selected from a group of 337 villages of west Bengal by SRSWOR.

Problem 02 ~ Following is a distribution of students in five classes of a school:

Class	Student Strength
I	45
II	32
III	27
IV	35
V	19

Draw a random sample of 10 students from the school.

Solution ~ The distribution of students in five classes of a school may be written as:

Class	Student strength	Serial number of students belonging to class
I	45	1 - 45
II	32	46 - 77
III	27	78 - 104
IV	35	105 - 139
V	19	140 - 158

Here, It is not specified whether to use SRSWR or SRSWOR, so, we should consider SRSWOR.

We shall take three-digited numbers from the table of random numbers. To ensure equal probability for each student, we shall take the numbers from 001 to 948 (The greatest three-digited multiple of 158) and shall ignore the other three digited numbers. We shall divide the number by 158 and take the remainder. The remainder varies from 000 to 157. The remainder 001 to 157 will be taken to correspond to the students with the same numbers, whereas the remainder 000 are correspond to 158 th student.

Since the sampling is without replacements, a student once selected cannot be selected again

Table~03

Number taken from the table Random no. (R).	Remainder when divided by 158 R(mod158)	Serial number of the student selected	Member selected
571	097	97	19th members of III.
173	015	15	15th memberof I.
437	121	121	17th members of IV
539	065	65	20th members of II
368	052	52	7th members of II
493	019	19	19th membersof I
975	Reject	—	—
335	019	— (Repeat)	—
403	087	87	10th members of III
114	114	114	10th members of IV
862	072	72	27th membesof II
588	114	— (Repeat)	—
330	014	14	14th members of I.

So a random sample of 10 students selected from the school are

19th student of class III,
15th student of class I,
17th student of class IV,
20th student of class II,
7th student of class II,
19th student of class I,
10th student of class III,
10th student of class IV,
27th student of class II,
14th student of class I.

Problem ~ 03 Draw a random sample of 7 days from a given leap year.

Solution ~ We shall take three-digited numbers from the table of random numbers. To ensure equal probability for each individual days, we shall take the numbers from 001 to 732 (The greatest three-digited multiple of **366**) and shall ignore the other three-digited numbers. We shall divide the numbers by 366 and take the remainder. The remainder, of course, varies from 000 to 365. The remainder 001 to 365 will be taken to correspond to the days with the same numbers, whereas the remainder 000 will corresponds to the **366** st day. Since we have not given any information whether the sampling is with replacements or without replacements, so we would consider the sampling is without replacement, a day once selected cannot be selected again. The selection is done in a tabular form as shown below:

Table ~ 04

Number taken from table (Random no. R)	Remainder when divided by 366 (R (mod 366))	Serial number of the day selected
991	Rejected	———
734	Rejected	———
905	Rejected	———
533	167	167
257	257	257
743	Rejected	———
480	114	114
971	Rejected	———
258	258	258
019	019	19
436	070	70
376	010	10

7 days with serial number 167, 257, 114, 258, 19, 70, 10 from a given leap year ($N = 366, n = 7$)

Problem 04 ~ For a Honours class of 16 students, the marks attained in paper I of Hons. Subject are:

67, 52, 84, 59, 30, 80, 67, 72, 55, 48, 59, 80, 39
67, 82, 52.

Select a random sample of 5 students from the class.

(i) Estimate the average marks.

(ii) Find the relative standard error and a estimate of it.

Solution ~ We shall take two-digited numbers from the table of random number. To ensure equal probability for each individual, we shall take numbers from 01 - 96 (The greatest two-digited multiple of 16) and shall ignore the other two-digited numbers. we shall divide the number by 16 and take the remainder. The remainder varies from 00 to 15. The remainder 01 to 15 will be taken to correspont to the marks of the student with the same serial numbers, whereas the remainder 00 will correspond to 16 th students' marks. Since the sampling is whether SRSWR or SRSWOR. We are not having any information about it. so, we would consider the sampling as without replacement. Since the sampling is without replacements, a student (corresponding marks in paper I) once selected cannot be selected again.

$\left.\begin{array}{l} N=16 \\ n=5 \end{array}\right\}$ 67, 52, 84, 59, 30, 80, 67, 72,
55, 48, 59, 80, 39, 67, 82, 52.

Table ~ 05

Number taken from the table Random no. (R)	Remainder when divided by 16 R (mod 16)	serial number of the student selected	Corresponding marks
13	13	13	39
49	01	1	67
04	04	4	59
17	01	——— (Repeat)	———
93	13	——— (Repeat)	———
11	11	11	59
97	Rejected	———	———
87	07	7	67

(i) A random sample of 5 students from a class selected with serial number 13, 1, 4, 11, 7 with corresponding marks as 39, 67, 59, 59, 67. Let Y_i denote the marks of αth student. $(i = 1(1)16)$.

We draw a SRSWOR sample of size $n = 5$.
Let us denote the marks of the ith student in paper 1 by y_i $(i = 1(1)5)$. [From selected sample students].

We know in SRSWOR, $\frac{1}{N}\sum_{\alpha=1}^{N} Y_\alpha$ (Population mean) is estimated by $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$.

$$\text{Here, } \bar{Y} = \frac{1}{N}\sum_{\alpha=1}^{16} Y_\alpha$$

$$= \frac{993}{16} = 62.025 \text{ is}$$

estimated by $\bar{y} = \frac{1}{n}\sum_{i=1}^{5} y_i$

so, Estimate of average marks $= 58.2$ | $= \frac{291}{5} = 58.2.$

ii) We know that the relative standard error

$$= \frac{\sqrt{var(\bar{y})}}{\bar{Y}}$$

For SRSWOR, $var(\bar{y}) = S_Y^2\left(\frac{1}{n} - \frac{1}{N}\right)$ | $\sum_{\alpha=1}^{N} Y_\alpha^2 = 65351$, $\bar{Y} = 62.0625$

$$S_Y^2 = \frac{1}{N-1}\sum_{\alpha=1}^{N}(Y_\alpha - \bar{Y})^2$$

$$= \frac{1}{(N-1)}\left[\sum_{\alpha=1}^{N} Y_\alpha^2 - N\bar{Y}^2\right] = \frac{1}{15}\left[65351 - 16\times(62.0625)^2\right]$$

$$= 248.1958333$$

$$Var(\bar{y}) = 248.1958333\left(\frac{1}{5} - \frac{1}{16}\right) = 34.12692708$$

$$\bigstar\ R.S.E = \text{relative standard error} = \frac{5.841825664}{62.025}$$

$$= 0.094185016.$$

Now, $var(\bar{y})$ is unbiasedly estimated by $s_y^2\left(\frac{1}{n} - \frac{1}{N}\right)$ in SRSWOR

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{4}\left[\sum_{i=1}^{5} y_i^2 - 5\bar{y}^2\right]$$

$$= 131.2$$

| $\sum_{i=1}^{5} y_i^2 = 17461$, $\bar{y} = 58.2$

estimate of $var(\bar{y}) = s_y^2\,(1/n - 1/N) = 131.2\left(\frac{1}{5} - \frac{1}{16}\right) = 18.04$

$\sqrt{var(\bar{y})} = 4.247352116$

$\bigstar$ $\therefore$ Estimate of rSE $= \frac{4.247352116}{58.2} = 0.072978558$

Problem 05 ~ Following table gives the scores of 45 students in the examination :

38	27	19	51	42	20	30	26	22
14	24	30	02	26	26	12	43	11
10	13	35	27	06	34	40	24	37
40	24	37	30	42	22	21	15	23
20	40	26	48	15	02	12	19	33.

Draw a random sample of size 15 students (a) With replacement (b) without replacement. In each case, first an estimate of the standard error of the sample mean.

Solution ~(a) We shall take two-digited numbers from the table of random number. To ensure equal probability for each individual students, we shall take the numbers from 01 - 90 (The greatest -two digited multiple of 45) and shall ignore the other 2-digited numbers. We shall divide the number by 45 and take the remainder. The remainder varies from 00 to 44. The remainder 001 to 44 will be taken to correspond to students score with the same serial numbers.
whereas the remainder 00 will correspond to the 45 th students score
Here, Since the sampling is with replacements, a student marks once selected can be selected again.

38	27	19	51	42	20	30	26	22
14	24	30	02	26	26	12	43	11
10	13	35	27	06	34	40	24	37
40	24	37	30	42	22	21	15	23
20	40	26	48	15	02	12	19	33

Table~06

Numbers from table Random no. (R)	Remainder when divided by 45. R(mod 45)	serial numbers of student selected	corresponding score
57	12	12	30
11	11	11	24
73	28	28	40
43	43	43	12
75	30	30	37

39	39	39	26
36	36	36	23
84	39	39	26
93	Rejected	—	—
97	Rejected	—	—
53	08	8	26
35	35	35	15
40	40	40	48
31	31	31	30
14	14	14	26
86	41	41	15
25	25	25	40

students with scores 30, 24, 40, 12, 37, 26, 23, 26, 26, 15, 48, 30, 26, 15, 40 are selected ramdomly with replacements.

(ii) We will follow the same process as above, Since the sampling is without replacements, A students score once selected cannot be selected again.

Table ~ 07

Number taken from table Random no.-(R)	Remainder when divided by 45. R(mod 45)	serial number of student selected	corresponding score
57	12	12	30
11	11	11	24
73	28	28	40
43	43	43	12
75	30	30	37
39	39	39	26
36	36	36	23
84	39	Repeat	—
93	Reject	—	—
97	Reject.	—	—

53	08	6	28
35	35	35	15
40	40	40	48
31	31	31	30
14	14	14	26
86	41	41	15
25	25	25	40
88	43	Repeat	22
33	33	33	

38	27	19	51	42	20	30	26	22
14	24	30	02	26	26	12	43	11
10	13	35	27	06	34	40	24	37
40	24	37	30	42	22	21	15	23
20	40	26	48	15	02	12	19	3

students with scores 30, 24, 40, 12, 37, 26, 23, 26, 15, 48, ~~18~~, 30, 26, 15, 40, 22 are selected randomly without replacements.

i) For the 1st case i.e. SRSWR, we have to find an estimate of the standard error of the sample mean.

$\therefore Var(\bar{y}) = \frac{\sigma_Y^2}{n}$ and this is unbiasedly estimated by $\frac{s_y^2}{n}$ in SRSWR.

so $S.E(\bar{y}) = \sqrt{Var(\bar{y})}$ is unbiasedly estimated by $\sqrt{\frac{s_y^2}{n}}$.

$$s_y^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (y_i - \bar{y})^2, \qquad \left[\sum_{i=1}^{15} y_i = 418 \right.$$

$$\bar{y} = 27.86667$$

$$s_y^2 = \frac{1}{(n-1)} \left[\sum_{i=1}^{n} y_i^2 - n\bar{y}^2 \right] \qquad \left. \sum_{i=1}^{15} y_i^2 = 13076. \right]$$

$$= \frac{1}{(14)} [13076 - 15 \times 27.86667^2]$$

$$= 101.9809524$$

Then $\sqrt{\frac{s_y^2}{n}} = \sqrt{6.79873016}$

$= 2.60743747$

• So, The estimate of the standard error of sample is 2.6074347 (SRSWR

ii) For SRSWOR case, We have to find an estimate of the standard error of the sample mean.

$\therefore$ $Var(\bar{y}) = S_Y^2\left(\frac{1}{n} - \frac{1}{N}\right)$ is unbiasedly estimated by $s_y^2\left(\frac{1}{n} - \frac{1}{N}\right)$.

so, standard error $(\bar{y}) = \sqrt{Var(\bar{y})}$ is unbiasedly estimated by $\sqrt{s_y^2\left(\frac{1}{n} - \frac{1}{N}\right)}$

Here $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$.

$s_y^2 = \frac{1}{(n-1)}\left[\sum_{i=1}^{n} y_i^2 - n\bar{y}\right]^2$

$\left[\sum_{i=1}^{15} y_i = 414, \ \bar{y} = 27.6\right.$

$\sum_{i=1}^{15} y_i^2 = 12884.$

$= \frac{1}{14}\left[12884 - 15(27.6)^2\right]$

$= 104.1142857.$

Now $\sqrt{s_y^2\left(\frac{1}{n} - \frac{1}{N}\right)} = \sqrt{10.4.1142857\left(\frac{1}{15} - \frac{1}{45}\right)}$

$= \sqrt{4.627301587}$

$= 2.151116358$

. so the estimate of the standard error of sample is 2.1511 (SRSWOR).

Problem 06 ~ Draw a random sample of size 8 from 231 iron balls having distribution of weight (in m.g.) as detailed below:

Weight	25.8	25.9	26.0	26.1	26.2	Total
Frequency	45	49	67	42	28	231

Solution ~ The iron balls having distribution of weight (in m.g) may be written as :-

Weight (in mg)	Frequency	Serial number of iron balls belonging to the weight class
25.8	45	1 - 45
25.9	49	46 - 94
26.0	67	95 - 161
26.1	42	162 - 203
26.2	28	204 - 231
Total	231	

We shall take three-digited numbers from the table of random numbers. To ensure equal probability to each iron balls we shall take the numbers from 001 to 924. (The greatest three digited multiple of 231) and shall ignore the other three-digited numbers. We shall divide the number by 231 and take the remainder. The remainder varies from 000 to 230. The remainder 001 to 230 will be taken to correspont to the iron balls with the same serial numbers, whereas the remainder 000 will correspond to the 231st iron ball. Since the information is not being provided here. whether It is SRSWR or SRSWOR we would consider the sampling without replacements. a iron ball once selected cannot be selected again

Here we have Table ~ 08

Number taken from the table Random no. (R)	Remainder when divided by 231 R(mod 231)	Serial number of the ball selected	corresponding ball selected with weights (in mg)
465	003	3	3rd ball of weight (25
238	007	7	7 th ball of wt (25.8
198	198	198	37th ball of wt (26.1
431	200	200	39th ball of wt (26.1
215	215	215	12 th ball of wt (26.2
023	023	23	23rd ball of wt (25.8
522	060	60	15 th ball of wt (25.9
472	010	10	10th ball of wt (25.9

A random sample of size 8 from 231 iron balls are selected without replacements are.

3rd iron ball of weight 25.8 mg.
7 th iron ball of weight 25.8 mg.
37th iron ball of weight 26.1 mg.
39th iron ball of weight 26.1 mg.
12th iron ball of weight 26.2 mg.
23 rd iron ball of weight 25.8 mg.
15 th iron ball of weight 25.9 mg.
10 th iron ball of weight 25.8 mg.

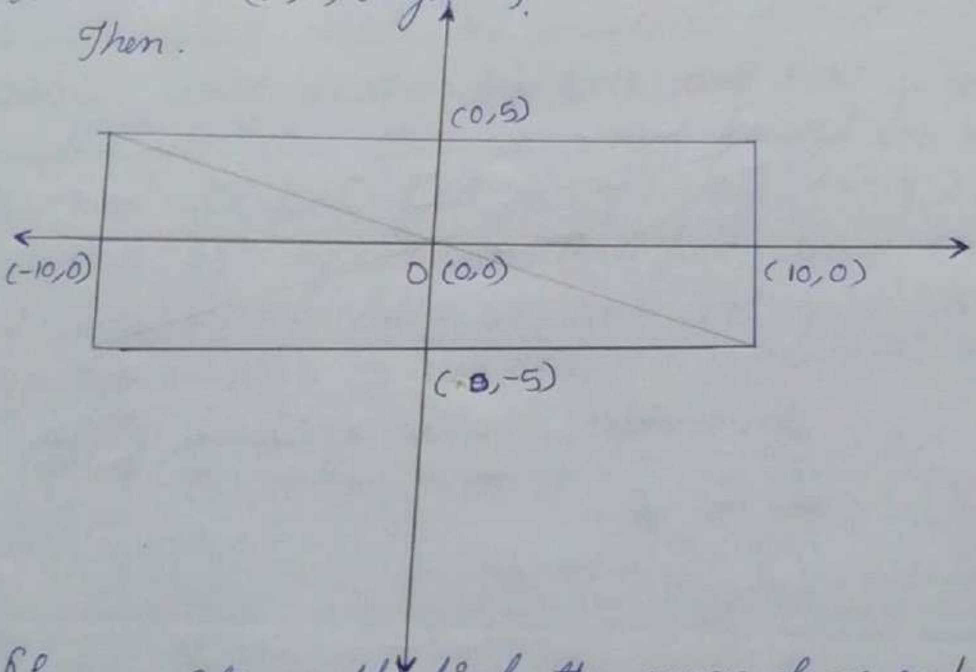
Problem 07 ~ Locate 7 random points in a rectangler area of size ___ × ___ sq meter [Use coordinate correct to cm].

Solution ~ Let length = 20 m = x
breadth = 10 m = y

$\therefore$ $xy = 200\ m^2$ = Area of the rectangle

We have to locate 7 random points in this rectangular area. (Let us consider that the diagonals of rectangle bisects each other at (0,0) origin).

Then.



Now, we could find the range of x and y as,

$$-10 \le x \le 10 \quad \text{and} \quad -5 \le y \le 5$$

$$\Rightarrow \quad 0 \le x+10 \le 20 \quad \text{and} \quad 0 \le y+5 \le 10$$

let $x+10 = x'$ & $y+5 = y'$.

$$0 \le x' \le 20 \ \underrightarrow{\ *\ } \ \& \quad 0 \le y' \le 10 \ \text{——} \ **$$

Now we are given that the coordinates could be correct upto cm It means It sould be 2 digits after decimal point

so multiply both sides of * & ** by 10^2.

$$\left.\begin{array}{l} 0 < 10^2 x' \le 2000 \\ 0 < 10^2 y' \le 1000 \end{array}\right\} \Rightarrow \begin{array}{l} 0 \le x'' \le 2000 \\ 0 \le y'' \le 1000 \end{array}$$

where $x'' = 10^2 x'$ and $y'' = 10^2 y'$.

x'' should lie within 0 and 2000, we have total 2001 choices and y'' should lie within 0 and 1000, we have total 1001 choices for y''.
We shall take 4-digit numbers from the table of random numbers. Every time we will take pairs of random numbers one from 0001 – 8004 (The greatest four-digit multiple of 2001 and one from 0001 – 9009 and will ignore all other four-digit numbers.
Then we shall divide the numbers (pairs of numbers) by 2001 and another by 1001 and take a unit less than the remainders the we would proceed in a reverse order, we will find first (x'', y'') then (x', y') by dividing x'' & y'' by 10^2 and then find pairs (x, y) by (x', y') relationship. Since we have not given any information whether to use SRSWR or SRSWOR, we will apply sampling without replacements.

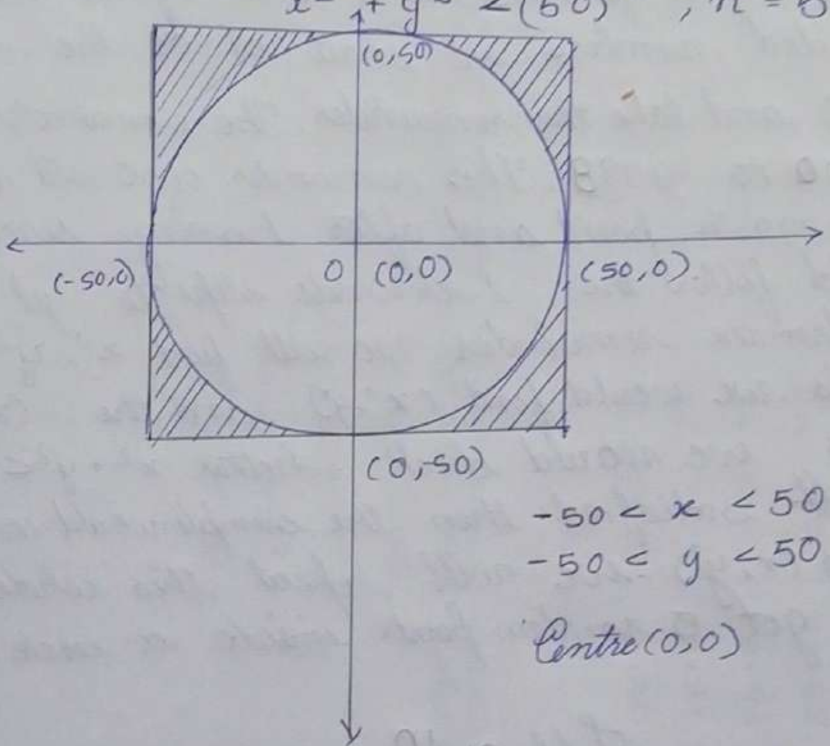
Table ~ 09

1st random number (R_1)	2nd random number (R_2)	$x'' = R_1 \pmod{2001} - 1$	$y'' = R_2 \pmod{1001} - 1$	$x' = x''/10^2$	$y' = y''/10^2$	$x = x' - 10$	$y = y' - 5$	Points Selected
4652	9031	649	Reject	—	—	—	—	
2030	0641	28	640	0.28	6.40	-9.72	1.40	(-9.72, 1.40)
8479	9917	Reject	Reject	—	—	—	—	—
6876	7287	372	279	3.72	2.79	-6.28	-2.21	(-6.28, -2.21)
0592	6499	591	492	5.91	4.92	-4.09	-0.08	(-4.09, -0.08)
0769	8678	768	669	7.68	6.69	-2.32	1.69	(-2.32, 1.69)
0178	3392	177	388	1.77	3.88	-8.23	-1.12	(-8.23, -1.12)
0264	4089	263	84	2.63	0.84	-7.37	-4.16	(-7.37, -4.16)
9376	3039	Reject	35	—	—	—	—	—
8971	0373	Reject	0372	—	—	—	—	—
9092	2464	Reject	461	—	—	—	—	—
8027	5754	1025	748	10.25	7.48	0.25	2.48	(0.25, 2.48)

Problem 08 ~ Plot 5 random points inside a circle of radius 50 cm. (coordinates are to be correct upto two places of decimals.)

Solution ~ We have to plot 5 random points inside a circle i.e

$$x^2 + y^2 < (50)^2, \quad n = 5.$$



$-50 < x < 50$
$-50 < y < 50$

Centre $(0,0)$

We would select those pairs (x,y) which satisfy $-50 < x < 50$, $-50 < y < 50$ as well as $x^2 + y^2 < (50)^2$.

$-50 < x < 50 \Rightarrow 0 < x + 50 < 100$

$-50 < y < 50 \Rightarrow 0 < y + 50 < 100$

so $0 < x' < 100$ —* where, $x + 50 = x'$
$0 < y' < 100$ —** $\quad y + 50 = y'$

Given that the coordinates are to be correct upto two places of decimals. For 2 digits after decimal we would multiply both the sides of * & ** by 10^2.

$0 < x' 10^2 < 10000 \Rightarrow 0 < x'' < 10^4$

$0 < y' 10^2 < 10000 \Rightarrow 0 < y'' < 10^4$

[where $x'' = 10^2 x'$ & $y'' = 10^2 y'$]

Both x'' and y'' should lie between 0 to 10^4 but except 0 and 10^4.

∴ We got total 9999 choices for both x" & y".
We shall take four-digited numbers from the table of random numbers. To ensure equal probability to each point, we shall take the numbers from 0001 to 9999 (highest 4-digited multiple of 9999) and shall ignore the other four-digited number. We shall divide the number by 9999 and take the remainder. The remainder varies from 0000 to 9998. The remainder 0000 will correspond to the 9999 th point and after having remainders we would follow the backwards steps to get (x, y). firstly from the remainder we will find x", y" then from this, we would find (x', y') and the (x, y) after this we would check whether $x^2 + y^2 < 2500$ or not, If satisfied then the sample would contain the pair (x, y). We will repeat this whole process till we get 5 random points inside a circle of radium 50 cm.

Table ~ 10

1st Random Numbers	2nd Random Number	$x'' = R_1 \pmod{9999}$	$y'' = R_2 \pmod{9999}$	$x' = x''/10^2$	$y' = y''/10^2$	$x = x' - 50$	$y = y' - 50$	$x^2+y^2 < 2500$ holds or not	Points selected
4652	3819	4652	3819	4652	38.19	-3.48	-11.81	Yes	(-3.48, -11.8
8431	2150	8431	2150	84.31	21.50	34.31	-28.50	Yes	(34.31, -28.50
2352	2472	2352	2472	23.52	24.72	-26.48	-25.28	Yes	(-26.48, -25.2
0043	3488	0043	3488	00.43	34.88	-49.57	-15.12	NO	———
9031	7617	9031	7617	90.31	76.17	40.31	26.17	Yes	(40.31, 26.17
1220	4129	1220	4129	12.20	41.21	-37.80	-8.79	Yes	(-37.80, -8.7

So, we plot 5 random points: (-3.48, -11.81) inside a circle of radius 50 cm.
(34.31, -28.50)
(-26.48, -25.28)
(40.31, 26.17)
(-37.80, -8.79)

Problem - 09 Draw a random sample 10 from the students for whome the following frequency distribution of marks in mathematics and statistics have been obtained:

Marks in statistics	Marks in mathematics					
	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
30 - 40	6	2	—	—	—	—
40 - 50	4	8	4	2	—	—
50 - 60	20	24	36	12	6	2
60 - 70	7	28	26	24	8	3
70 - 80	—	8	16	20	6	6

Solution ~ We have 6 groups of marks in mathematics and 5 groups of marks in statistics.
Overall we have 6 × 5 = 30 groups of marks. and out of them 7 groups does not contain any individual.

Here, • cell (1, 1) contains all the students with 40-50 marks in mathematics and 30-40 marks in statistics.

• cell (1, 2) contains all those students who secured 50-60 marks in mathematics and 30-40 marks in statistics

⋮

• cell (5, 6) contains all those students who attained 90-100 marks in mathematics and 70-80 marks in statistics.

We can write the distribution as.

Serial number	Members
1 - 6	Members of cell (1,1)
7 - 8	Members of cell (1,2)
9 - 12	Members of cell (2,1)
13 - 20	Members of cell (2,2)
21 - 24	Members of cell (2,3)

25 - 26	Members of cell (2,4)
27 - 46	Members of cell (3, 1)
47 - 70	Members of cell (3,2)
71 - 106	Members of cell (3,3)
107 - 118	Members of cell (3,4)
119 - 124	Members of cell (3,5)
125 - 126	Members of cell (3,6)
127 - 133	Members of cell (4,1)
134 - 161	Members of cell (4,2)
162 - 187	Members of cell (4,3)
188 - 211	Members of cell (4,4)
212 - 219	Members of cell (4,5)
220 - 222	Members of cell (4,6)
223 - 230	Members of cell (5,2)
231 - 246	Members of cell (5, 3)
247 - 266	Members of cell (5, 4)
267 - 272	Members of cell (5,5)
273 - 278	Members of cell (5,6)

N = 278
n = 10.

We shall take three digited numbers from the table of random numbers. To ensure equal probability to each individual, we shall take the numbers from 001 to 834 (The greatest three-digited multiple of 278) and shall ignore the other three digited numbers. We shall divide the numbers by 278 and take the remainder. The remainder varies from 000 to 277. The remainders 001 to 277 will be taken to correspond to the students with the same series numbers, whereas the remainder 000 will correspond to 278 th student. Since we are not given any information whether to apply SRSWR or SRSWOR. We will use SRSWOR, Since the sampling is without replacements, a student once selected cannot be selected again.

Random number (R)	Remainder when divided by 278	Serial number of student selected	Members selected
571	015	15	3rd member of (2,2)
173	173	173	12th member of (4,3)
437	159	159	26th member of (4,2)
539	261	261	15th member of (5,4)
368	090	90	20th member of (3,3)
493	215	215	4th member of (4,5)
975	Rejected	———	———
335	057	57	11th member of (3,2)
403	125	125	1st member of (3,6)
148	148	148	15th member of (4,2)
625	069	69	23rd member of (3,2)

Table~11

So, 3rd member of (2,2), 12th member of (4,3), 26th member of (4,2), 15th member of (5,4), 20th member of (3,3), 4th member of (4,5), 11th member of (3,2), 1st member of (3,6), 15th member of (4,2), 23rd member of (3,2). are the selected random sample of size 10 without replacements.

Problem 10 ~ Draw a random sample of size n= _ from a binomial (m= , p=) distribution.

Solution ~ we will draw a random sample of size $n=5$ from Bin ($m = 10$, $p = 0.75$) distribution

We know that Binomial random variable with parameters (m,p)
= Number of success in a sequence of m bernoulli-trials with probability of success p per trial.

Here, $p = 0.75$. If we select 2 digit random numbers then we may call it a success if 00 – 74 occurs & failure otherwise.

We have Success : 00 − 74 → 75 two digit random numbers
Failure : 75 − 99 → 25 two digit random numbers

$$P(S) = \frac{\text{No. of success}}{\text{Total number of trials}} = \frac{75}{100} = 0.75$$

We select $m = 10$, 2 digit random number and calculate how many of them are lying between 00 − 74.

This number is the number of success i.e. binomial random variable.

We need to repeat this process $n = 5$ times to get 5 random samples.

Random numbers	lies between 00−74 or not
46	yes
52	yes
38	yes
19	yes
84	No
31	yes
21	yes
50	yes
23	yes
52	yes

Number of successes = 9

Random numbers	lies between 00−74 or not
24	yes
72	yes
00	yes
43	yes
34	yes
88	No
90	No
31	yes
76	No
17	yes

Number of successes = 7

Random numbers	lies between 00-74 or not
12	yes
20	yes
41	yes
29	yes
71	yes
48	yes
19	yes
43	yes
49	yes
90	No

Numbers of successes = 9

Random No.	whether lie between 00-74 or not
17	yes
49	yes
20	yes
30	yes
23	yes
27	yes
73	yes
53	yes
60	yes
07	yes

Number of success = 10

Random Number	whether lie between 00-74 or not
94	No
10	yes
91	No
79	No
27	yes
22	yes
84	No
45	yes
06	yes
41	yes

Number of success = 6

So, a random sample of size n = 5 from a binomial (m = 10, p = 0.7)
is 9, 7, 9, 10 6

Problem 11 ~ Draw a random sample of size from a poisson (λ =) distribution.

Solution ~ We will draw a random sample of size 10 from Poisson distribution with (λ = 4) distribution.

We shall take three-digit numbers from the table of random variable and we will try to assign numbers to each group for each value of x using the distribution function or also cumulative probability. after assign groups from 000 to 999, we will check each 3 digit random number If The random number lies in any of the group, Then the corresponding value will be a sample we will repeat this 10 times to get 10 random sample.

We know that the random variable X is said to follow Poisson distribution with parameter λ i.e. $X \sim Poi(\lambda)$ If It has the pmf.

$$P(X=x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & \text{for } x = 0,1,2,\cdots ; \lambda > 0 \\ 0 & \text{, otherwise} \end{cases}$$

and
The distribution function is

$$F(x) = \begin{cases} 0 & \text{If } x < 0 \\ P(X=0) & \text{If } 0 \le x < 1 \\ P(X=0) + P(X=1) & \text{If } 1 \le x < 2 \\ \vdots & \\ \text{so on} & \end{cases}$$

$$P(X=x) = \frac{e^{-4}4^x}{x!}$$

$e^{-4} = 0.018315638$ (Taking upto 3 decimal places)
$e^{-4} = 0.018$.

so,

$P(X=0) = 0.018$
$P(X=1) = 0.072$
$P(X=2) = 0.144$
$P(X=3) = 0.192$
$P(X=4) = 0.192$
$P(X=5) = 0.154$
$P(X=6) = 0.102$
$\vdots$

x	$P(X=x)$	$P(X \le x)$ or Cumulative probability	Assigning range of groups
0	0.018	0.018	0.000 – 0.017
1	0.072	0.090	0.018 – 0.089
2	0.144	0.234	0.090 – 0.233
3	0.192	0.426	0.234 – 0.425
4	0.192	0.618	0.426 – 0.617
5	0.154	0.772	0.618 – 0.771
6	0.102	0.874	0.772 – 0.873
7+	0.126	1.000	0.874 – 0.999

Now we will pick up random number then divide them by 1000, The range in which it would lie the corresponding x will be a sample. [3-digit random numbers]

Random numbers	Random numbers/1000	Corresponding range	Corresponding x
247	0.247	0.234 - 0.425	3
730	0.730	0.618 - 0.771	5
002	0.002	0.000 - 0.017	0
700	0.700	0.618 - 0.771	5
078	0.078	0.018 - 0.089	1
002	0.002	0.000 - 0.017	0
292	0.292	0.234 - 0.425	3
293	0.293	0.234 - 0.425	3
361	0.361	0.234 - 0.425	3
252	0.252	0.234 - 0.425	3

As $0 < U \leq F(0) \Rightarrow$ sample $\rightarrow 0$

$F(0) < U \leq F(1) \Rightarrow$ sample $\rightarrow 1$

so, a random sample of size 10 from a Poisson ($\lambda = 4$) distribution is 3, 5, 0, 5, 1, 0, 3, 3, 3, 3.

Problem ~12 Select a random sample of size 10 from a $N(\mu = \quad, \sigma = \quad)$ distribution.

Solution We will select a random sample of size 10 from $N(\mu = 20, \sigma = 10)$ distribution with the help of Box-Muller transformation as a box Muller transform takes a continuous, two dimensional uniform distribution and transforms it to a normal distribution.

Lets say U_1 and U_2 are original independent uniform random variables i.e. The two variables are uniformly distributed in the interval (0, 1), The Box-Muller transform creates Z_1 and Z_2; independent, random variables that have a standard normal distribution

$$Z_1 = \sqrt{-2\ln U_1}\cos(2\pi U_2)$$

$$Z_2 = \sqrt{-2\ln U_1}\sin(2\pi U_2)$$

After obtaining a random sample of size 10 from standard normal population $N(0,1)$. To obtain a random sample from $N(\mu = 20, \sigma = 10)$ population we convert the Z- values obtained to X - values by the relation:

$$Z = \{(X - \mu)/\sigma\}$$

$$\Rightarrow X = \mu + \sigma Z$$

or $z_1 = \{(x_1 - \mu)/\sigma\}$ & $z_2 = \{(x_2 - \mu)/\sigma\}$

$\Rightarrow x_1 = \mu + \sigma z_1$ & $x_2 = \mu + \sigma z_2$

First for selecting uniform random sample we will take 3-digit random number (without any rejection) and then divide it by 10^3 and If we get 000 as random number then 1.000 will be the uniform random sample.

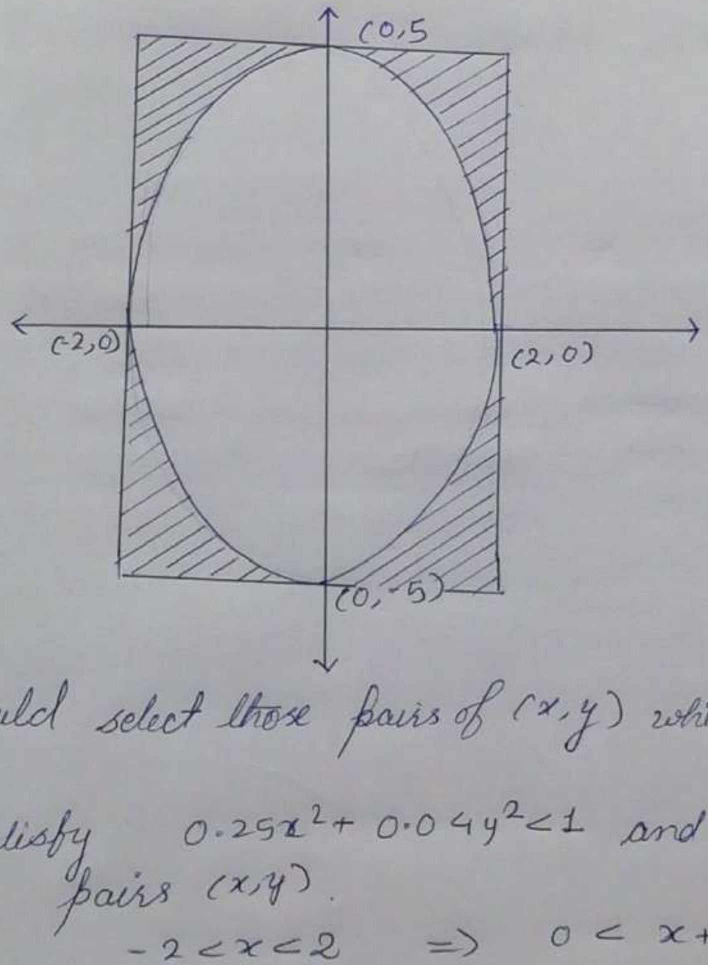
1st random Number	2nd Random number	U_1	U_2	z_1	z_2	x_1	x_2
909	247	0.909	0.247	0.008	0.436	20.08	24.36
730	002	0.730	0.002	0.793	0.010	27.93	20.10
700	078	0.700	0.078	0.745	0.398	27.45	23.98
002	292	0.002	0.292	-0.920	3.403	10.80	54.03
293	361	0.293	0.361	-1.006	1.201	9.94	119.4

Thus the x - values: (20.08, 24.36, 27.93, 20.10, 27.45, 10.80, 23.98, 54.03, 9.94, 119.4) constitute the random sample of size 10 from $N(20, 10^2)$ population.

Problem~13 Insert 9 random point (Coordinates in cm.) within the area bounded by the inequality $0.25x^2+0.04y^2<1$, both x and y being measured in meters.

Solution~ We have to plot 9 random points (coordinates in cm within the area bounded by the ellipse $0.25x^2+0.04y^2<1$ (both x and y being measured in meters).

Let we have the ellips $0.2x^2+0.04y^2=1$ with $(0,0)$ as centre then $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$ (equation of ellipls.



we would select those pairs of (x,y) which are $-2<x<2$
$-5<y<5$

and satisfy $0.25x^2+0.04y^2<1$ and we would reject all other pairs (x,y).

$-2<x<2 \Rightarrow 0<x+2<4$

$-5<y<5. \Rightarrow 0<y+5<10$

$0<x'<4$ —* where $x'=x+2$

$0<y'<10$ —* $y'=y+5$

Give that coordinates are to be correct upto cm (correct upto two places of decimals) For correct upto two digits after decimals we would multiply both the sides of * * & * by 10^2

$0<x'10^2<400 \Rightarrow 0<x''<400$ $\big\}$ $x''=10^2x')$

$0<y'10^2<1000 \Rightarrow 0<y''<1000$ $\big\}$ $y''=10^2y'$.

x" should lie from 0 to 400 [Except 0 & 400].
we got total 399 choices for x" and
y" should lie from 0 to 1000 [Except 0 & 1000],
we got total 999 choices for y".

Each time we will draw 3-digit random number pairs
we shall take numbers from 001 to 798. (highest three
digit multiple of 399).
& another time we well take numbers from 001 to 999
(highest 3 digit multiple of 999). and we shall ignore
the other three digit numbers.
The we will proceed to draw sample in the following
tabular form.

1st Random Number	2nd Random Number	$x'' = R_1 (\mathrm{mod}\ 899)$	$y'' = R_2 (\mathrm{mod}\ 999)$	$x' = \frac{x''}{10^2}$	$y' = \frac{y''}{10^2}$	$x = x' - 2$	$y = y' - 5$	$\frac{x^2}{4}+\frac{y^2}{25}<1$ holds or not	Point selected
134	904	134	904	1.34	9.04	−0.66	4.04	yes	(−0.66, 4.0
179	311	179	311	1.79	3.11	−0.21	−1.89	yes	(−0.21, −1.8
978	712	Reject	712	—	—	—	—	—	—
840	769	Reject		—	—	—	—	—	—
842	210	Reject	210	—	—	—	—	—	—
774	234	375	234	3.75	2.34	1.75	−2.66	No	—
024	877	024	877	0.24	8.77	−1.76	3.77	No	—
606	504	207	504	2.07	5.04	0.07	0.04	yes	(0.07, 0.04
275	440	275	440	2.75	4.40	0.75	−0.60	yes	(0.75, −0.6
842	908	Reject	908	—	—	—	—	—	—
068	803	068	803	0.68	8.03	−1.32	3.03	Yes	(−1.32, 3.03)
201	704	201	704	2.01	7.04	0.01	2.04	Yes	(0.01, 2.04
436	575	037	575	0.37	5.75	−1.63	0.75	Yes	(−1.63, 0.7
263	037	263	037	2.63	0.37	0.63	−4.63	Yes	(0.63, −4.6
475	636	076	636	0.76	6.36	−1.24	1.36	Yes	(−1.24, 1.3

9 random points within the area bounded by the inequality $0.25x^2 + 0.04y^2 < 1$, both x and y being measured in meters are $(-0.66, 4.04)$, $(-0.21, -1.89)$, $(0.07, 0.04)$, $(0.75, -0.60)$ $(-1.32, 3.03)$, $(0.01, 2.04)$, $(-1.63, 0.75)$ $(0.63, -4.63)$ $(-1.24, 1.36)$ (in meters) or we can say $(66, 404)$ $(21, 189)$, $(7, 4)$, $(75, 60)$, $(-132, 303)$, $(1, 204)$, $(-163, 75)$ $(63, -463)$, $(-124, 136)$ in cms.