Example 1 : semi Averages

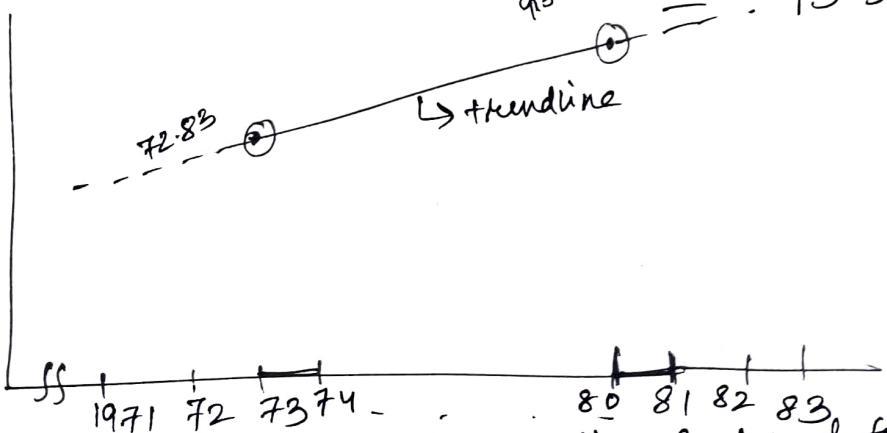
1. Fit a trend line to the following data by method of semi-averages.

Year	Bank clearance
1971	53
72	79
73	76
74	66
75	69
76	94
77	105
78	87
79	79
80	104
81	97
82	92
83	101

Hint. n = no. of time points = 13 (odd)
Omit middle one that is $t = 7$ & 7

From 71–76 mean of bank clearance $= \frac{53 + \cdots + 94}{6}$
$= 72.83$

From 77–83 " " " " $\frac{105 + 87 + \cdots + 101}{6}$
$= 93.33$



Now you extend it to find the fitted trend for other year point and also for forecasting the future value.

Question : Forecast for year = 1985.
the trend

Example 2

Below is the population of Indian census from 1901 – 1971.

Year (t)	Population (U_t)	$t' = \frac{t - 1936}{5}$
1901	238.3	−7
1911	252.0	−5
1921	251.2	−3
1931	278.9	−1
1936 → 1941	361.0	1
1951	439.1	3
1961	328.5	5
1971	547.9	7

Question

(1) Compute and a straight line trend with the observed data.

(2) Compute a 2nd degree polynomial. Also plot it using excel.

(3) Eliminate linear trend considering multiplicative model.

Example 3

Fit an exponential trend $U_t = ab^t$ to the following data by method of least squares and find the trend values. Estimate the population in 1981.

Census (t):	1911	1921	1931	1941	1951	1961	1971
Population (in crores):	25.0	25.1	27.9	31.9	36.9	43.9	54.7.

Hint: First transform t. (& t 1936)

$$t' = \frac{t - 1936}{5}$$

Fit the modified exponential equation to the following data.

Year :	1981	1982	1983	1984	1985	1986
Production :	81	89	98	109	120	133

Hint. The modified exponential equation is

$$U_t = a + bc^t$$

split in three equal parts

$t = 1, 2$; $t = 3, 4$; $t = 5, 6$.

$$S_1 = \sum_{t=1}^{2} U_t =$$

$$S_2 = \sum_{3}^{4} U_t =$$

$$S_3 = \sum_{5}^{6} U_t = \ldots$$

Then complete it.

Ex. Do the same taking exponential trend.

year (*)	Production	$t' = \frac{t - 1983.5}{1.5}$	Trend values.
1981	81	-5	
1982	89	-3	
1983	98	-1	
1984	109	1	
1985	120	3	
1986	133	5	

$$y = ab^t$$

$$\Rightarrow \log y = \log a + t \log b$$

$$\Rightarrow V = A + Bt$$

By Least square principle $A = \frac{\Sigma V}{n}$, $B = \frac{\Sigma tV}{\Sigma t^2}$

Now find the exponential equation.

Ex. Given the three selected points U_1, U_2 and U_3 corresponding to $t_1 = 2$, $t_2 = 30$ and $t_3 = 58$ as follows.

$t_1 = 2 \quad U_1 = 55.8$

$t_2 = 30 \quad U_2 = 138.6$

$t_3 = 58 \quad U_3 = 251.8$

Fit logistic curve by the method of selected points. Also obtain the trend values for t = 5, 18, 50, 70