B.Sc. (Honours) Examination, 2020
Semester-III
Statistics
Course: CC 5
(Sampling Distribution)
Time: 3 Hours   Full Marks: 40

Questions are of value as indicated in the margin
Notations have their usual meanings

Answer **any four** questions

1. 1. a. Show that $X_{(k)}$ in a random sample of size n from a $R(0,1)$ distribution has a
    beta distribution with parameters $(k, n-k+1)$.
    b. Show that the pdf of the sample range from an $R(0,1)$ distribution is given by
    $n(n-1)r^{n-2}(1-r), 0 \le r \le 1, r(range) = x_{(n)} - x_{(1)}$.   5+5

2.   a. Write down the test procedure to perform a large sample test for comparing two
    independent binomial proportions.
    b. Hence or otherwise find a $100(1-\alpha)\%$ confidence interval for the difference of
    proportions. Find the expected length of the interval.   6+4

3. a. Let X_1 and X_2 be independently binomially distributed random variables, with
    parameters $(n_1, \frac{1}{2})$ and $(n_2, \frac{1}{2})$, respectively. Show that $X_1 - X_2 + n_2$ has the
    binomial distribution with parameters $(n_1 + n_2, \frac{1}{2})$.
    b. Let X and Y be independently distributed, each in the form $N(0,1)$. Show that
    $Z = X/Y$ has the Cauchy distribution with pdf
    $$f(z) = \frac{1}{\pi[1+z^2]}$$
    What would be the distributions of $W_1 = X/|Y|$ and $W_2 = X/|X|$?   4+6

4. (a) Let X_1 and X_2 be two independently distributed random variables following $R(0,1)$
    distributions. Then find the distributions of the following two random variables:
    $$U_1 = \sqrt{-2lnX_1} \cos 2\pi X_2 , U_2 = \sqrt{-2lnX_1} \sin 2\pi X_2$$
    (b) State and prove WLLN for Bernoulli variable.

5. (a) Define χ^2 distribution. Find its mean and variance. Prove the additive property of
    this distribution.
    (b) State and prove DeMoivre-Laplace Central Limit theorem (CLT).   5+5

6. (a) Derive the pdf of an F-distribution.
   (b) If X and Y are independent random variables each distributed uniformly over $(0,1)$,
       find the distributions of
       (i) $\frac{X}{Y}$ (ii) XY and (iii) $\sqrt{X^2 + Y^2}$ 3+7