BSc (Honours) Semester -III Examination 2020
Subject- Statistics
Paper- CC6A (Statistical Inference-Theory)

Full Marks: 40 Time: 3 hours

Answer any four questions: (Notations have usual meanings)

1. What do you understand by Point Estimation? Define the following terms and give one example for each: Sufficient Statistic, Unbiased Estimator, Consistent Estimator, Efficient Estimator. 10

2. a) Show that if T is an unbiased estimator of a parameter θ, then $\lambda_1 T + \lambda_2$ is an unbiased estimator of $\lambda_1\theta + \lambda_2$, where λ_1 and λ_2 are known constants, but T^2 is a biased estimator of θ^2. 6

b) Let T_n be an estimator of θ with variance σ_n^2 and $E(T_n) = \theta_n$. Prove that if $\theta_n \to \theta$ and $\sigma_n^2 \to 0$, as $n \to \infty$ then T_n is a consistent estimator of θ. 4

3. a) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population with pdf

$$f(x,\theta) = \theta\, e^{-\theta x}\ ;\ x > 0,\ \theta > 0$$

Find Cramer-Rao lower bound for the variance of the unbiased estimator of θ. 4

b) State Neyman-Pearson Lemma for testing simple versus simple hypothesis. If $x \geq 1$ is the critical region for testing H_0: $\theta=2$ against the alternative H_1: $\theta=1$, on the basis of a single observation from the population

$$f(x,\theta) = \theta\, x^{\theta-1},\ \text{if } 0 < x < 1$$
$$= 0, \quad \text{otherwise}$$

where $0 < \theta < \infty$.
Obtain the values of type-I and type-II errors and power function of the test. 6

4. a) Let $x_1, x_2, \ldots, x_n$ be a random sample from the Bernoulli population with parameter θ, $0 < \theta < 1$. Obtain a sufficient statistic for θ and show that it is complete. Hence find minimum variance unbiased estimator (MVUE) of θ. 7

b) $x_1, x_2, \ldots, x_{10}$ is a random sample of size 10 from a Poisson distribution with mean λ. Show that the critical region W defined by $\sum_{i=1}^{10} x_i \geq 3$, is the best critical region for testing H_0: $\lambda=0.1$ against the alternative H_1: $\lambda=0.5$. 3

5. What are simple and composite statistical hypotheses? Give examples. Explain the following terms in the context of testing of statistical hypothesis:
Most Powerful Test, Uniformly Most Powerful Test, Power function of a test, Level of significance. 10

6. a) An urn contains 6 marbles of which θ are white and others are black. In order to test the null hypothesis H_0: $\theta=3$ against the alternative H_1: $\theta=4$, two marbles are drawn at random (without replacement) and H_0 is rejected if both the marbles are white; otherwise H_0 is accepted. Find the probabilities of committing type-I and type-II errors. 5

b) Given a random sample $x_1, x_2, \ldots, x_n$ of size n from the distribution with pdf

$$f(x,\theta) = \theta\, e^{-\theta x}\ ;\ x > 0,\ \theta > 0$$

show that UMP test for testing H_0: $\theta = \theta_0$ against H_1: $\theta < \theta_0$ is given by $W = \{\mathbf{x}: \sum x_i \geq (1/2\theta_0)\chi^2_{\alpha,\,2n}\}$. 5