B.Sc. (Honours) Examination, 2021
Semester-V
Statistics
Course: CC-11
(Stochastic Process and Queuing Theory (Theory))

Time: 3 Hours **Full Marks: 40**

Questions are of value as indicated in the margin

Notations have their usual meanings

Group – A (Answer any four questions) $4 \times 10 = 40$

1. a) X and Y are iid $Bin(n,p)$ RVs i.e. pmf $P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, \cdots, n$. Then using MGF find the pmf of $Z = X+Y$. 5
   b) Write short notes: Classification of one-dimensional stochastic processes, Pure death processes. 5

2. a) Show that a Markov chain is completely defined by initial and transitional probabilities. 5
   b) Suppose there are three popular online shopping sites (Amazon, Flipkart and Myntra) have lots of customer. Among them 20%, 15% change their preferences from Flipkart to Amazon, Myntra respectively in every month; 24%, 18% change their preferences from Myntra to Amazon, Flipkart respectively in every month and 12%, 9% change their preferences from Amazon to Flipkart, Myntra respectively in every month. Let X_n be the preference of the customer in n month. Then for the MC $\{X_n\}$, find its one and two step transition probability matrix. 5

3. a) The TPM of a MC $\{X_n,\ n = 1, 2, \cdots\}$ having four states $1, 2, 3, 4$ and 5 is

$$P = \begin{array}{c} X_{n+1}= \\ X_n=1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0 & 0.3 & 0.2 \\ 0.2 & 0 & 0.3 & 0 & 0.5 \\ 0 & 0.3 & 0.4 & 0.3 & 0 \\ 0.2 & 0 & 0.3 & 0 & 0.5 \\ 0.3 & 0.3 & 0 & 0.4 & 0.1 \end{bmatrix} \end{array}.$$

   Represent the transition probability matrix as a stochastic graph. 2
   b) In usual notations, show that $p_{ij}^{(\alpha)} = \sum_{r=1}^{m} p_{ir}^{(\alpha-\beta)} p_{rj}^{(\beta)} = \sum_{r=1}^{m} p_{ir}^{(\beta)} p_{rj}^{(\alpha-\beta)}$, where the SP having states $1, 2, \cdots, m$ and α is a positive integer. 5
   c) Let $\{X_n, n \geq 0\}$ be a MC has three states $1, 2, 3$ with TPM is $P = \begin{pmatrix} 0.3 & 0.4 & 0.3 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.7 & 0.1 \end{pmatrix}$ and initial distribution $P(X_0 = i) = \frac{1}{3}$, $i = 1, 2, 3$. Find the two and three step TPMs. Hence find $P(X_4 = 1 \mid X_2 = 2)$, $P(X_8 = 3 \mid X_6 = 2)$. 3

4. a) Write short notes on: Persistent and Transient State, Recurrence time distribution. 5
   b) If $X(t)$ is a Poission process then find the auto-correlation coefficient between $X(t)$ and $X(t+s)$. 5

5. a) Explain M/M/1 model with the basic assumptions? In usual notations, show that $p_n = \rho^n(1-\rho)$ for M/M/1 model. Here ρ is utilization factor. 8
   b) Suppose that customers arrive at a service counter in accordance with a Poisson process with mean rate of 2 per minute. Then find the probability of two successive arrivals between 1 to 2 minutes. 2

6. Consider a sequence of random variable $\{X_n,\ n \geq 1\}$ such that each of X_n assumes only two values -1 and 1 with conditional probabilities with TPM, $P = \begin{matrix} {\scriptstyle X_{n+1}=} \\ {\scriptstyle X_n=-1} \\ {\scriptstyle 1} \end{matrix} \begin{matrix} \\ \left[\begin{matrix} \overset{-1}{1-\alpha} & \overset{1}{\alpha} \\ \beta & 1-\beta \end{matrix} \right] \end{matrix}$. If X_n denotes the direction of movement to the left or, right corresponding to the value -1 or, 1 respectively at the n^{th} step. The initial distribution $P(X_0 = 1) = p_0$, $P(X_0 = -1) = q_0 = 1 - p_0$. Show that the probability of event occurs in all trials are same at i.e. $P(X_n = 1) = p_n = \frac{\alpha}{\alpha+\beta} + (p_0 - \frac{\alpha}{\alpha+\beta})(1-\alpha-\beta)^{n-1}$. Hence find $EX_n,\ V(X_n),\ E(X_nX_{n-1}),\ E(X_nX_{n-2})$. 10